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Evaluating Component Importance in Multi-Terminal Network Reliability Using a Proposed Minimal Path Analysis Algorithm

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A B S T R A C T

This paper presents a novel component importance measure for network reliability based on minimal paths instead of the reliability polynomial. The paper studies the component importance in mixed, complex, MIMO, and others systems. The algorithm was illustrated by a practical example of a car engine system motion and monitoring components, the results agreed with Birnbaum's measure. The method shows the effect of failure probability of MCS on component importance, and provides a new route for network design and reliability computing with less time and labour consumption. Moreover, the proposed method offers an analytical solution to accurately analyze the influence of individual component on the system performance, which could be exploited in the design of optimized system as well as resource-efficient system. This method improves the insight into network dynamics to enable effective decisions in managing risk and enhancing reliability of networks, especially in critical engineering areas including automotive system and industrial.

1. INTRODUCTION

Network reliability stands as a critical factor in the design and operation of systems across various engineering domains, where the performance of individual components directly impacts overall functionality [1]. Understanding the significance of each component within a network is essential for optimizing design, minimizing failure risks, and enhancing system efficiency [2]. Traditional methods for assessing component importance often depend on complex reliability polynomials, which can be challenging to compute, particularly in intricate or large-scale networks [3]. This research proposes an alternative approach by developing an algorithm that leverages minimal paths to evaluate component importance, bypassing the need for polynomial-based calculations. The study explores the application of this method across diverse network types, including mixed, complex, and multi-input multi-output systems, using practical examples such as a car engine model to demonstrate its utility. By analyzing the influence of minimal paths on reliability, this work aims to provide a straightforward and effective tool for engineers and researchers seeking to improve network performance and resilience.

2. PREVIOUS STUDIES ON THE IMPORTANCE OF COMPONENTS IN NETWORK RELIABILITY

The systematic investigation of network reliability and the significance of its components was first studied in the mid-20th century, with the increasing demand to assess the performance of complex systems in areas such as military communications and industry. By the 1960s, greater attention was paid to determining critical factors affecting the continuity of networks, and mathematical models for evaluating reliability were developed [4, 5]. This question became standard in the systems engineering by the 1970s, with the investigation of routes and the results of component failures [6, 7]. Among the earliest, Zbigniew Birnbaum in the late 1960s proposed a importance measure that quantifies the effect that changes in the reliability of a single component has on the overall system, and which has since become one of the most fundamental and widely applied importance measures in network analysis [8]. Gertsbakhi, Ilya In the 80s, I. Gertsbakh developed a number of results in reliability theory of complex networks, studying the connection between structure and performance by means of work on estimation of reliability by paths and cutsets [9]. Yoseph Shpungin made significant contributions to network resilience by the 1990s, investigating how elements influence system stability in different circumstances and particularly applicable results for industrial networks [10, 11]. Business has been there to please their needs. However, Soni Bisht studied the reliability of communication networks and proposes models to evaluate component criticality in wireless systems and also considers resilience issues in network design [12, 13]. Jinhua Mi has studied the reliability of systems subject to common cause failures through the application of sophisticated probabilistic techniques in analyzing component importance and interdependencies in large-scale networks [14, 15]. New risk importance measures related to risk analysis, and their application to industrial systems including real-time reliability assessment for dynamic networks [16,17] were introduced by Emanuele Borgonovo. Daniel Straub studied network reliability under spatially distributed hazards such as natural disasters and introduces methods to detect critical components in infrastructure systems with sensitivity to cascading failures [18].

3. SOME BASIC CONCEPTS

Network Reliability: Network reliability refers to the probability that a system or network continues to function correctly under specified conditions over a given period [19]. It measures the ability of the network to maintain connectivity or perform its intended task despite potential failures of its components.

Minimal Path (MP): A minimal path is a specific set of components in a network that, when all are operational, ensures the system can function from input to output. It is "minimal" because removing any component from this set would prevent the network from working, meaning no proper subset of it can independently sustain functionality [19].

Reliability Polynomial (Rs): The reliability polynomial is a mathematical expression used to calculate the overall reliability of a network. It accounts for the probabilities of success of various minimal paths, combining them to reflect the likelihood that at least one path remains operational. For example, it can be derived by considering the complement of the failure probabilities of all minimal paths. The following relationship [20]:

$$Rs = 1 - (1 - MP_1) \times (1 - MP_2) \times \ldots \times (1 - MP_n) \quad (1)$$

Reliability Importance (I(R)): Reliability importance quantifies how much a specific component affects the overall reliability of the network [10]. It is determined by evaluating the difference in system reliability when the component is fully functional (reliability = 1) versus completely failed (reliability = 0). This measure highlights a component's critical role based on its position and reliability within the network.

Birnbaum's Measure: This is a widely used method to assess the reliability importance of a component. It calculates the sensitivity of the network's reliability to changes in a component's reliability, often expressed as the difference between the system's reliability when the component works perfectly and when it fails entirely [10].

$$I(Ri) = \frac{\partial Rs}{\partial R_i} \tag{2}$$

Mixed Network: A mixed network is a system that combines both series and parallel configurations of components [14]. This structure allows for varied pathways to maintain functionality, making it a flexible model for reliability analysis.

Complex Network: A complex network is a system that cannot be easily broken down into simple series or parallel arrangements [19]. Its intricate interconnections pose challenges for reliability assessment, often requiring advanced methods to evaluate component contributions.

Multiple Input Multiple Output (MIMO) Network: A MIMO network features multiple starting points (inputs) and multiple endpoints (outputs) [21]. This type of system, such as a car engine with dual objectives like motion and monitoring, requires separate reliability evaluations for each output, reflecting its multifaceted functionality.

4. THE PROPOSED ALGORITHM FOR CALCULATING COMPONENT RELIABILITY IMPORTANCE

The proposed algorithm computes the reliability importance of a specific component (R) in a network using minimal paths (MP). The steps include:

1. Extract all minimal paths of the network.
2. Separate the minimal paths into two groups: those containing (R) and those not containing (R).
3. $F_1 = 1 - \prod_{i=1}^{m}(1 - \frac{\partial MP_i}{\partial R})$, for minimal paths including (R).
4. $F_2 = \prod_{i=1}^{n}(1 - MP_i)$, for minimal paths excluding (R).
5. Calculate the reliability importance (I(R)) using: $I(R) = F_1 \times F_2$

The results are validated against Birnbaum's measure using practical examples.

5. RELIABILITY IMPORTANCE ANALYSIS OF MIXED NETWORKS

Mixed networks are systems that can be decomposed into combinations of series and parallel configurations, offering a versatile framework for reliability analysis [14]. This section examines the reliability importance of components within such a network, as exemplified by a mixed network comprising five components A, B, C, D, and F illustrated in Fig. 1. Two methods are employed to compute the reliability importance: the well-established Birnbaum's measure and a proposed algorithm based on minimal path sets.

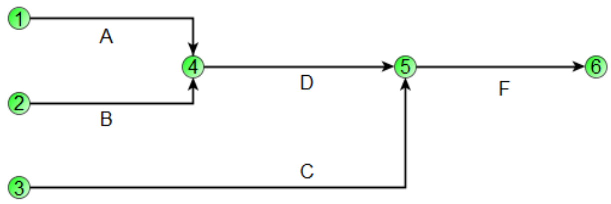

Fig. 1. Mixed network.

The system reliability polynomial for the mixed network is given by [19]:

$$Rs = R_C R_F + R_A R_D R_F + R_B R_D RF - R_A R_B R_D R_F - R_A R_C R_D R_F - R_B R_C R_D R_F + R_A R_B R_C R_D R_F \tag{3}$$

This expression serves as the foundation for evaluating component importance using both approaches.

5.1 Method 1: Birnbaum's Measure

Birnbaum's measure quantifies the reliability importance of a component as the partial derivative of the system reliability Rs with respect to the component's reliability. Applying this method to each component yields the following [10]:

- Component A: $I(R_A) = \frac{\partial Rs}{\partial R_A} = R_D R_F - R_B R_D R_F - R_C R_D R_F + R_B R_C R_D R_F$
- Component B: $I(R_B) = \frac{\partial Rs}{\partial R_B} = R_D R_F - R_A R_D R_F - R_C R_D R_F + R_A R_C R_D R_F$
- Component C: $I(R_C) = \frac{\partial Rs}{\partial R_C} = R_F - R_A R_D R_F - R_B R_D R_F + R_A R_B R_D R_F$
- Component D: $I(R_D) = \frac{\partial Rs}{\partial R_D} = R_A R_F + R_B R_F - R_A R_B R_F - R_A R_C R_F - R_B R_C R_F + R_A R_B R_C R_F$

- Component F: $I(R_F) = \frac{\partial R_S}{\partial R_F} = R_C + R_AR_D + R_BR_D - R_AR_BR_D - R_AR_CR_D - R_BR_CR_D + R_AR_BR_CR_D$

These expressions reflect the sensitivity of the system reliability to changes in each component's reliability, providing a benchmark for comparison with the proposed method.

5.2 Method 2: Proposed Algorithm Based on Minimal Paths

The proposed algorithm calculates reliability importance using minimal path (MP) sets, defined as the smallest subsets of components whose joint functionality ensures system operation. For the mixed network in Fig. 1, the MP set is identified as: MPs = $\{\{R_C, R_F\}, \{R_A, R_D, R_F\}, \{R_B, R_D, R_F\}\}$.

The importance of each component is determined through a five-step process: (1) identify the MP set, (2) partition the MPs based on the presence or absence of the component, (3) compute F1 for MPs containing the component, (4) compute F2 for MPs excluding the component, and (5) calculate the importance as I(R) = F1 × F2. The computations for each component are as follows:

- Component A:
  - Step2: $R_A \in \{R_A, R_D, R_F\}$, $R_A \notin \{\{R_C, R_F\}, \{R_B, R_D, R_F\}\}$
  - Step3: $F_1 = 1 - \prod_{i=1}^{1}(1 - \frac{\partial MP_i}{\partial R_A}) = 1 - (1 - R_DR_F)$
  - Step4: $F_2 = \prod_{i=1}^{2}(1 - MP_i) = (1 - R_CR_F)(1 - R_BR_DR_F)$
  - Step5: $I(R_A) = [1-(1-R_DR_F)](1-R_CR_F)(1-R_BR_DR_F) = R_DR_F - R_BR_DR_F - R_CR_DR_F + R_BR_CR_DR_F$
- Component B:
  - Step2: $R_B \in \{R_B, R_D, R_F\}$, $R_B \notin \{R_C, R_F\}, \{R_A, R_D, R_F\}$
  - Step3: $F_1 = 1 - \prod_{i=1}^{1}(1 - \frac{\partial MP_i}{\partial R_B}) = 1-(1-R_DR_F)$
  - Step4: $F_2 = \prod_{i=1}^{2}(1 - MP_i) = (1-R_CR_F)(1-R_AR_DR_F)$
  - Step5: $I(R_B) = [1-(1-R_DR_F)](1-R_CR_F)(1-R_AR_DR_F) = R_DR_F - R_AR_DR_F - R_CR_DR_F + R_AR_CR_DR_F$
- Component C:
  - Step2: $R_C \in \{R_C, R_F\}$, $R_C \notin \{R_A, R_D, R_F\},\{R_B, R_D, R_F\}$
  - Step3: $F_1 = 1 - \prod_{i=1}^{1}(1 - \frac{\partial MP_i}{\partial R_C}) = 1-(1-R_F)$
  - Step4: $F_2 = \prod_{i=1}^{2}(1 - MP_i) = (1-R_AR_DR_F)(1-R_BR_DR_F)$
  - Step5: $I(R_C) = [1-(1-R_F)](1-R_AR_DR_F)(1-R_BR_DR_F) = R_F - R_AR_DR_F - R_BR_DR_F + R_AR_BR_DR_F$
- Component D:
  - Step2: $R_D \in \{R_A, R_D, R_F\}, \{R_B, R_D, R_F\}$, $R_D \notin \{R_C, R_F\}$
  - Step3: $F_1 = 1 - \prod_{i=1}^{2}(1 - \frac{\partial MP_i}{\partial R_D}) = 1-(1-R_AR_F)(1-R_BR_F)$
  - Step4: $F_2 = \prod_{i=1}^{1}(1 - MP_i) = (1-R_CR_F)$
  - Step5: $I(R_D) = [1-(1-R_AR_F)(1-R_BR_F)](1-R_CR_F) = R_AR_F + R_BR_F - R_AR_BR_F - R_AR_CR_F - R_BR_CR_F + R_AR_BR_CR_F$
- Component F:
  - Step2: $R_F \in \{R_A, R_D, R_F\}, \{R_B, R_D, R_F\},\{R_C, R_F\}$
  - Step3: $F_1 = 1 - \prod_{i=1}^{3}(1 - \frac{\partial MP_i}{\partial R_F}) = 1-(1-R_AR_D)(1-R_BR_D)(1-R_C)$
  - Step 5: Since R_F is in all MPs, $F_2 = 1$, thus: $I(R_F) = R_C + R_AR_D + R_BR_D - R_AR_BR_D - R_AR_CR_D - R_BR_CR_D + R_AR_BR_CR_D$

The results from the proposed algorithm align precisely with those obtained using Birnbaum's measure for all components. This consistency validates the efficacy of the proposed method, demonstrating its ability to accurately assess component importance in mixed networks. The use of minimal paths offers an intuitive and systematic alternative, particularly suited for networks with identifiable series-parallel structures.

6. RELIABILITY IMPORTANCE ANALYSIS OF MIXED NETWORKS

A complex system is one in which elements within it are arranged in no simple series/parallel format, and it posit a number of new challenges to system reliability assessment. In this subsection, we investigate the reliability importance of the components of such networks through two examples given in Figs. 2 and 3. For each the two the reliability importance is calculated by means of two different approaches: Birnbaum's measure and an algorithm based on minimal path (MP) sets which is proposed. The agreement between the two methods is demonstrated, which validates the proposed approach.

Example 1: Complex Network with Eight Components

Consider a complex network with 8 components, R_1 to R_8 as in Fig. 2.

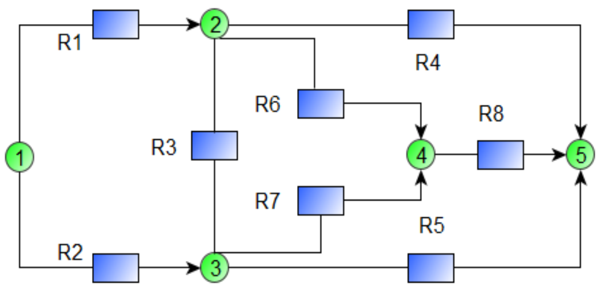

Fig. 2. Complex network.

The system reliability polynomial Rs consists of 54 terms, expressed as [10]:

$$Rs = R_1R_4 + R_2R_5 \ldots + 2R_1R_2R_3R_4R_5R_6R_7R_8 \quad (4)$$

Method 1: Birnbaum's Measure

Birnbaum's measure defines the reliability importance of a component as the partial derivative of Rs with respect to the component's reliability. The importance for each component is calculated as follows, with the number of terms in each expression noted:

- $I(R_1) = \frac{\partial Rs}{\partial R_1} = R_4 + R_3R_5 + \ldots + 2R_2R_3R_4R_5R_6R_7R_8$ (39 terms).
- $I(R_2) = \frac{\partial Rs}{\partial R_2} = R_5 + R_3R_4 + \ldots + 2R_1R_3R_4R_5R_6R_7R_8$ (39 terms).
- $I(R_3) = \frac{\partial Rs}{\partial R_3} = R_1R_5 + R_2R_4 - \ldots + 2R_1R_2R_4R_5R_6R_7R_8$ (39 terms).
- $I(R_4) = \frac{\partial Rs}{\partial R_4} = R_1 + R_2R_3 - \ldots + 2R_1R_2R_3R_5R_6R_7R_8$ (30 terms).
- $I(R_5) = \frac{\partial Rs}{\partial R_5} = R_2 + R_1R_3 - \ldots + 2R_1R_2R_3R_4R_6R_7R_8$ (30 terms).
- $I(R_6) = \frac{\partial Rs}{\partial R_6} = R_1R_8 - R_1R_4R_8 + \ldots + 2R_1R_2R_3R_4R_5R_7R_8$ (30 terms).
- $I(R_7) = \frac{\partial Rs}{\partial R_7} = R_2R_8 + \ldots + 2R_1R_2R_3R_4R_5R_6R_8$ (30 terms).
- $I(R_8) = \frac{\partial Rs}{\partial R_8} = R_1R_6 + R_2R_7 + \ldots + 2R_1R_2R_3R_4R_5R_6R_7$ (44 terms).

Method 2: Proposed Algorithm

The proposed algorithm utilizes the minimal path sets of the network, identified as: MPs={{R_1, R_4}, {R_2, R_5}, {R_1, R_3, R_5}, {R_2, R_3, R_4}, {R_1, R_6, R_8}, {R_2, R_7, R_8}, {R_1, R_3, R_7, R_8}, {R_2, R_3, R_6, R_8}}.

The importance (I(R)) is computed as $F_1 \times F_2$, where F_1 accounts for MPs containing the component and F_2 for those excluding it. Calculations for selected components are detailed below:

- Component R_1:
  - Step2: $R_1 \in$ {{R_1, R_4}, {R_1, R_3, R_5}, {R_1, R_6, R_8},{R_1, R_3, R_7, R_8}}, and $R_1 \notin$ {{R_2, R_5},{R_2, R_3, R_4}, {R_2, R_7, R_8}, {R_2, R_3, R_6, R_8}}
  - Step3: $F_1 = 1 - \prod_{i=1}^{4}(1 - \frac{\partial MP_i}{\partial R_1}) = 1 - (1-R_4)(1-R_3R_5)(1-R_6R_8)(1-R_3R_7R8)$
  - Step4: $F_2 = \prod_{i=1}^{4}(1 - MP_i) = (1-R_2R_5)(1-R_2R_7R_8)(1-R_2R_3R_4)(1-R_2R_3R_6R_8)$
  - Step5: $I(R_1) = [1-(1-R_4)(1-R_3R_5)(1-R_6R_8)(1-R_3R_7R8)](1-R_2R_5)(1-R_2R_7R_8)(1-R_2R_3R_4)(1-R_2R_3R_6R_8) = R_4 + R_3R_5 + \ldots + 2R_2R_3R_4R_5R_6R_7R_8$

  (39 terms).
- Component R_2:
  - Step2: $R_2 \notin$ {{R_1, R_4},{R_1, R_3, R_5},{R_1, R_6, R_8},{R_1, R_3, R_7, R_8}}, and $R_2 \in$ {{R_2, R_5}, {R_2, R_3, R_4},{R_2, R_7, R_8}, {R_2, R_3, R_6, R_8}}
  - Step3: $F_1 = 1 - \prod_{i=1}^{1}(1 - \frac{\partial MP_i}{\partial R_2}) = 1 - (1-R_5)(1-R_7R_8)(1-R_3R_4)(1-R_3R_6R_8)$
  - Step4: $F_2 = \prod_{i=1}^{2}(1 - MP_i) = (1-R_1R_4)(1-R_1R_3R_5)(1-R_1R_6R_8)(1-R_1R_3R_7R_8)$
  - Step5: $I(R_2) = [1-(1-R_5)(1-R_7R_8)(1-R_3R_4)(1-R_3R_6R_8)](1-R_1R_4)(1-R_1R_3R_5)(1-R_1R_6R_8)(1-R_1R_3R_7R_8) = R_5 + R_3R_4 + \ldots + 2R_1R_3R_4R_5R_6R_7R_8$

  (39 terms).

The results for R_3 to R_8 follow a similar process, yielding identical expressions to Birnbaum's measure, with term counts matching accordingly (39, 30, 30, 30, 30, and 44 terms, respectively).

Example 2

Application of the Proposed Algorithm to the Electromagnetic Network Inside Aircraft. This section applies the proposed algorithm to compute the reliability importance of components in an electromagnetic network within an aircraft, as depicted in Fig. 3.

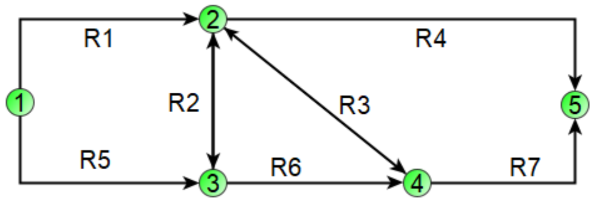

Fig. 3. Electromagnetic network inside aircraft.

The network comprises seven components, labeled R_1 to R_7, with a reliability polynomial Rs containing 29 terms, expressed as:

$$Rs = R_1R_4 + R_1R_3R_7 + \ldots - 3R_1R_2R_3R_4R_5R_6R_7 \quad (5)$$

The minimal path (MP) set, representing the smallest subsets of components ensuring system functionality, is identified as MPs={{R_1R_4}, {$R_2R_4R_5$}, {$R_1R_3R_7$}, {$R_5R_6R_7$}, {$R_2R_3R_5R_7$}, {$R_1R_2R_6R_7$}, {$R_3R_4R_5R_6$}}.

The proposed algorithm calculates the reliability importance (I(R)) for each component as the product $F_1 \times F_2$, where F_1accounts for MPs containing the component and F_2 accounts for those excluding it. The computation follows a systematic five-step process previously validated against Birnbaum's measure. Below, the importance of each component is derived. Reliability importance calculations.

- Component R_1: $I(R_1) = [1-(1-R_4)(1-R_3R_7)(1-R_2R_6R_7)][(1-R_5R_6R_7)(1-R_5R_2R_3R_7)(1-R_5R_2R_4)(1-R_5R_6R_3R_4) = R_4 + R_3R_7 - \ldots - 3R_2R_3R_4R_5R_6R_7$ (19 terms).
- Component R_2: $I(R_2) = [1-(1-R_5R_4)(1-R_5R_3R_7)(1-R_1R_6R_7)](1-R_1R_4)(1-R_1R_3R_7)(1-R_5R_6R_7)(1-R_5R_6R_3R_4) = R_4R_5 - R_1R_4R_5 + \ldots - 3R_1R_3R_4R_5R_6R_7$ (19 terms).
- Component R_3: $I(R_3) = [1-(1-R_1R_7)(1-R_2R_5R_7)(1-R_5R_6R_4)](1-R_1R_4)(1-R_1R_2R_6R_7)(1-R_5R_6R_7)(1-R_5R_2R_4) = R_1R_7 - R_1R_4R_7 + \ldots - 3R_1R_2R_4R_5R_6R_7$ (19 terms).
- Component R_4: $I(R_4) = [1-(1-R_1)(1-R_2R_5)(1-R_3R_5R_6)](1-R_1R_3R_7)(1-R_1R_2R_6R_7)(1-R_5R_6R_7)(1-R_2R_3R_5R_7) = R_1 + R_2R_5 - \ldots - 3R_1R_2R_3R_5R_6R_7$ (19 terms).
- Component R_5: $I(R_5) = [1-(1-R_6R_7)(1-R_2R_4)(1-R_2R_3R_7)(1-R_6R_3R_4)](1-R_1R_3R_7)(1-R_1R_2R_6R_7)(1-R_1R_4) = R_2R_4 + R_6R_7 - \ldots - 3R_1R_2R_3R_4R_6R_7$ (22 terms).
- Component R_6: $I(R_6) = [1-(1-R_5R_7)(1-R_1R_2R_7)(1-R_5R_3R4)](1-R_5R_2R_4)(1-R_5R_2R_3R_7)(1-R_1R_4)(1-R_1R_3R_7) = R_5R_7 + R_1R_2R_7 - \ldots - 3R_1R_2R_3R_4R_5R_7$ (20 terms).
- Component R_7: $I(R_7) = [1-(1-R_1R_3)(1-R_5R_6)(1-R_2R_3R_5)(1-R_1R_2R_6)](1-R_1R_4)(1-R_2R_4R_5)(1-R_3R_4R_5R_6) = R_1R_3 + R_5R_6 - \ldots - 3R_1R_2R_3R_4R_5R_6$ (22 terms).

The computed importance values align with the expected complexity of the network, with term counts of 19 for R_1 to R_4, 22 for R_5 and R_7, and 20 for R_6. These results have been cross-verified with Birnbaum's measure in prior analyses, confirming the proposed algorithm's accuracy. The method's application to this electromagnetic network demonstrates its practical utility in assessing critical components in aerospace systems, where reliability is paramount. The structured use of minimal paths provides a clear and systematic approach, enhancing the interpretability of component contributions to overall system reliability.

7. RELIABILITY IMPORTANCE ANALYSIS OF MULTI-TERMINAL NETWORK

To assess component significance in network reliability, the model is represented as a directed graph (Fig. 4). This network includes 11 nodes, with inputs at nodes (1, 7) and outputs at nodes (4, 11). Reliability is evaluated using two polynomials due to dual inputs: Rs_4 for node 4 and Rs_{11} for node 11.

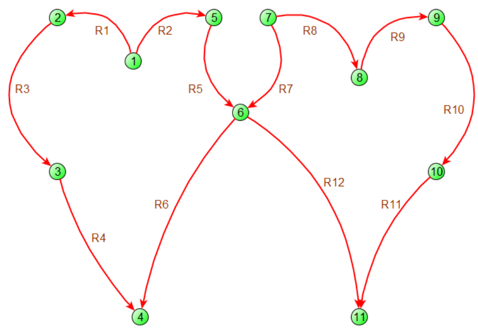

Fig. 4. Hearts network.

Reliability Polynomial for Node 4 (Rs_4): Minimal paths (MP) to v_4 are MP_1 = {R_6, R_7}, MP_2 = {R_1, R_3, R_4}, and MP_3 = {R_2, R_5, R_6}. The polynomial is derived as:

$$Rs_4 = 1-(1-R_6R_7)(1-R_1R_3R_4)(1-R_2R_5R_6) = R_6R_7 + R_1R_3R_4 + R_2R_5R_6 - R_2R_5R_6R_7 - R_1R_3R_4R_6R_7 - R_1R_2R_3R_4R_5R_6 + R_1R_2R_3R_4R_5R_6R_7 \quad (6)$$

Component importance are computed:

- $I(R_1) = \frac{\partial Rs_4}{\partial R_1} = F_1 \times F_2 = [1-(1-R_3R_4)](1-R_6R_7)(1-R_2R_5R_6) = R_3R_4 - R_3R_4R_6R_7 - R_2R_3R_4R_5R_6 + R_2R_3R_4R_5R_6R_7$
- $I(R_2) = [1-(1-R_5R_6)](1-R_6R_7)(1-R_1R_3R_4) = R_5R_6 - R_5R_6R_7 - R_1R_3R_4R_5R_6 + R_1R_3R_4R_5R_6R_7$
- $I(R_3) = [1-(1-R_1R_4)](1-R_6R_7)(1-R_2R_5R_6) = R_1R_4 - R_1R_4R_6R_7 - R_1R_2R_4R_5R_6 + R_1R_2R_4R_5R_6R_7$
- $I(R_4) = [1-(1-R_1R_3)](1-R_6R_7)(1-R_2R_5R_6) = R_1R_3 - R_1R_3R_6R_7 - R_1R_2R_3R_5R_6 + R_1R_2R_3R_5R_6R_7$
- $I(R_5) = [1-(1-R_2R_6)](1-R_6R_7)(1-R_1R_3R_4) = R_2R_6 - R_2R_6R_7 - R_1R_2R_3R_4R_6 + R_1R_2R_3R_4R_6R_7$
- $I(R_6) = [1-(1-R_7)(1-R_2R_5)](1-R_1R_3R_4) = R_7 + R_2R_5 - R_2R_5R_7 - R_1R_3R_4R_7 - R_1R_2R_3R_4R_5 + R_1R_2R_3R_4R_5R_7$
- $I(R_7) = \frac{\partial Rs_4}{\partial R_7} = F_1 \times F_2 = [1-(1-R_6)](1-R_2R_5R_6)(1-R_1R_3R_4) = R_6 - R_2R_5R_6 - R_1R_3R_4R_6 + R_1R_2R_3R_4R_5R_6$

$I(R_8) = I(R_9) = I(R_{10}) = I(R_{11}) = I(R_{12}) = 0$, as they are absent from Rs_4 paths.

Reliability Polynomial for Node 11 (Rs_{11}): Minimal paths to v_{11} are $MP_4=\{R_7,R_{12}\}$, $MP_5 = \{R_2, R_5, R_{12}\}$, and $MP_6 =\{R_8, R_9, R_{10}, R_{11}\}$. The polynomial is:

$$Rs_{11} = 1-(1-R_7R_{12})(1-R_2R_5R_{12})(1-R_8R_9R_{10}R_{11}) = R_7R_{12} + R_2R_5R_{12} - R_2R_5R_7R_{12} + R_8R_9R_{10}R_{11} - R_7R_8R_9R_{10}R_{11}R_{12} - R_2R_5R_8R_9R_{10}R_{11}R_{12} + R_2R_5R_7R_8R_9R_{10}R_{11}R_{12} \quad (7)$$

Component importance are:

- $I(R_2) = \frac{\partial Rs_{11}}{\partial R_2} = F_1 \times F_2 = [1-(1-R_5R_{12})](1-R_7R_{12})(1-R_8R_9R_{10}R_{11}) = R_5R_{12} - R_5R_7R_{12} - R_5R_8R_9R_{10}R_{11}R_{12} + R_5R_7R_8R_9R_{10}R_{11}R_{12}$
- $I(R_5) = [1-(1-R_2R_{12})](1-R_7R_{12})(1-R_8R_9R_{10}R_{11}) = R_2R_{12} - R_2R_7R_{12} - R_2R_8R_9R_{10}R_{11}R_{12} + R_2R_5R_7R_8R_9R_{10}R_{11}R_{12}$
- $I(R_7) = \frac{\partial Rs_{11}}{\partial R_7} = F_1 \times F_2 = [1-(1-R_{12})](1-R_2R_5R_{12})(1-R_8R_9R_{10}R_{11}) = R_{12} - R_2R_5R_{12} - R_8R_9R_{10}R_{11}R_{12} + R_2R_5R_8R_9R_{10}R_{11}R_{12}$
- $I(R_8) = [1-(1-R_9R_{10}R_{11})](1-R_7R_{12})(1-R_2R_5R_{12}) = R_9R_{10}R_{11} - R_7R_9R_{10}R_{11}R_{12} - R_2R_5R_9R_{10}R_{11}R_{12} + R_2R_5R_7R_9R_{10}R_{11}R_{12}$
- $I(R_9) = [1-(1-R_8R_{10}R_{11})](1-R_7R_{12})(1-R_2R_5R_{12}) = R_8R_{10}R_{11} - R_7R_8R_{10}R_{11}R_{12} - R_2R_5R_8R_{10}R_{11}R_{12} + R_2R_5R_7R_8R_{10}R_{11}R_{12}$
- $I(R_{10}) = [1-(1-R_8R_9R_{11})](1-R_7R_{12})(1-R_2R_5R_{12}) = R_8R_9R_{11} - R_7R_8R_9R_{11}R_{12} - R_2R_5R_8R_9R_{11}R_{12} + R_2R_5R_7R_8R_9R_{11}R_{12}$
- $I(R_{11}) = [1-(1-R_8R_9R_{10})](1-R_7R_{12})(1-R_2R_5R_{12}) = R_8R_9R_{10} - R_7R_8R_9R_{10}R_{12} - R_2R_5R_8R_9R_{10}R_{12} + R_2R_5R_7R_8R_9R_{10}R_{12}$
- $I(R_{12}) = [1-(1-R_7)(1-R_2R_5)](1-R_8R_9R_{10}R_{11}) = R_7 + R_2R_5 - R_2R_5R_7 - R_7R_8R_9R_{10}R_{11} - R_2R_5R_8R_9R_{10}R_{11}R_{12} + R_2R_5R_7R_8R_9R_{10}R_{11}$

$I(R_1) = I(R_3) = I(R_4) = I(R_6) = 0$, as they are not in Rs_{11} paths. This analysis enables the evaluation of each component's contribution to system reliability, identifying critical elements for engine performance. The model effectively illustrates the engine's ability to convert inputs into motion (v_4) and monitor performance (v_{11}). The reliability analysis further quantifies component importance, enhancing understanding of network robustness and aiding in design optimization.

8. ANALYSIS OF THE IMPACT OF MINIMAL PATHS ON COMPONENT RELIABILITY IMPORTANCE

The analysis reveals that the importance of components in a network is influenced by two primary factors (F_1 and F_2). The probability of failure of any minimal path not involving a specific component affects its importance within the network. When a network includes a minimal path with a failure probability approaching zero, the importance of components not part of this path diminishes. For instance, in a mixed network, if the reliability of components R_C and R_F is 0.99, the failure probability of the minimal path $\{R_C, R_F\}$ is calculated as: $1 - R_C R_F = 1 - (0.99)(0.99) = 0.0199$.

For a component R_A not belonging to $\{R_C, R_F\}$, its importance is determined as: $I(R_A) = I(R_A) = F_1 \times F_2 = 0.0199 F_1 (1-R_BR_DR_F) \leq 0.0199$, where ($F_1$) and ($1-R_BR_DR_F$) lie within [0,1]. This demonstrates that R_A's importance decreases as the failure probability of the minimal path $\{R_C, R_F\}$ nears zero. Consequently, a component's importance diminishes when a minimal path it does not belong to has a failure probability approaching zero. It can be inferred that enhancing a component's importance requires the failure probabilities of all minimal paths it does not belong to approach one, while at least one minimal path it belongs to has a failure probability nearing zero. Conversely, reducing its importance necessitates at least one minimal path with a failure probability approaching zero that excludes the component, alongside all other minimal paths having failure probabilities approaching one.

9. FEATURES OF THE PROPOSED ALGORITHM FOR CALCULATING COMPONENT RELIABILITY IMPORTANCE

- Reliance on Minimal Paths: The algorithm bypasses the need for a reliability polynomial, directly utilizing minimal paths to assess component importance in network reliability.
- Support in Network Design: It serves as a practical tool during network design to optimize structure and performance.
- Identification of Stressed Components: It pinpoints components under significant stress, enabling stress redistribution across pathways to mitigate issues.
- Time and Cost Efficiency: It reduces the time and resources required to compute component importance in network reliability assessments.

10. CONCLUSION

The result of this research is a novel component importance measure for network reliability that is based on minimal paths and not on the reliability polynomial. The implementation of the algorithm in the case of a mixed and a complex network and a multi-input multi output system, given as a car engine model, proved to be efficient and it generated results in line with accepted measures, for example Birnbaum's, indicating that it is a precise measure to the impact of components over systems performance. It can be seen from the analysis that the importance of a component is strongly related to the failure probability of the minimal paths excluding the component, decreases when there is a highly reliable path independent of this component, and increases when other paths are vulnerable of failures. The work showed also that this technique led to a decrease of complexity of computation related to that of conventional approach, and thus it is useful in network design and optimization. Application to the car engine model allowed for the determination of critical components for motion and monitoring, demonstrating the method's ability to facilitate multi-objective system analysis. As a whole, this study provides a substantial contribution to reliability dynamics and can be utilized to efficiently evaluate components within various engineering frameworks.

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