

Revival and Development of Heiser–Pratt Hypersonic Propulsion Models Utilizing Python: A Comparative and Modular Examination of Scramjet Burner and Performance Modules

Amr Abbassa,*

^a Department of Mechanical Engineering, Mississippi State University, Mississippi, USA.

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* Corresponding author:

Amr Abbass 
E-mail:
calgary732@hotmail.com

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ABSTRACT

This paper introduces the foundational validated Python reimplementation of the renowned Hypersonic Airbreathing Propulsion (HAP) modeling framework developed by Heiser and Pratt, focusing on trajectory analysis, burner design, and thrust performance modules. The expanded program reinstates the original HAP technique through modular code, an intuitive GUI, and improved flexibility to model both constant-pressure and constant-area scramjet combustion processes. A trajectory estimate module, utilizing atmospheric and inlet compression circumstances, calculates pre-burner states (station 3), whilst a generalized one-dimensional flow solver simulates heat addition and area influences within the combustor. The Python model results demonstrate remarkable concordance with HAP outputs: the burner exit Mach number aligns within 0.17%, static pressure and temperature ratios within 0.7%, and thrust parameters vary by less than 0.2% in constant pressure mode. Moreover, the Python tool incorporates constant-area combustion features that were lacking in the original HAP, rendering it particularly advantageous for parametric analyses, educational purposes, and conceptual design. This study fills a significant research void by providing a rapid, transparent, and validated substitute for the traditional HAP system, facilitating wider use and expansion for contemporary hypersonic propulsion research.

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1. INTRODUCTION

The modeling of high-speed airbreathing engines is essential for the progression of hypersonic

flight technology. The scramjet, functioning under supersonic combustion circumstances, poses distinct thermodynamic issues that require efficient, precise, and flexible simulation

techniques. For decades, the Hypersonic Airbreathing Propulsion textbook by Heiser and Pratt has supplied the fundamental equations for one-dimensional analysis of scramjet components, utilized in their heritage DOS HAP code. Nevertheless, the original program is inaccessible primarily to contemporary academics owing to obsolete platforms, restricted adaptability, and the absence of open-source availability.

A significant research gap persists due to the lack of a proven, user-friendly, and expandable tool that accurately mimics the HAP architecture. Current literature regularly references HAP; yet there are few, if any, attempts that provide a clear reimplementa-tion that aligns with its numerical outcomes. This work develops a comprehensive Python-based solver that replicates HAP's analytical approach while enhancing accessibility through GUI design, multiple combustion assumptions (constant pressure and constant area), and a clear modular structure.

This study has three primary objectives: (1) to reconstruct the original HAP trajectory, burner, and performance modules in Python utilizing precise one-dimensional flow equations; (2) to validate the model outputs against benchmark HAP data with high accuracy; and (3) to offer a lightweight, extensible platform for scramjet cycle design, education, and preliminary optimization.

2. LITERATURE REVIEW

The fundamental idea of hypersonic airbreathing propulsion is meticulously articulated by Heiser and Pratt [1], providing the analytical framework for scramjet engine design and evaluation. Roberts and Wilson [2] utilized these principles in their study of a scramjet engine with a transition Mach number of 4.0. Lim et al. [3] investigated inlet layouts, emphasizing shape transformation and compactness. Chakir [4] performed quasi-one-dimensional scramjet cycle modeling using the Heiser–Pratt methodology. Capistrano and Toro [5] presented the performance parameters of Brazil's VHA 14-XB system. Sarosh [6] delineated an extensive scramjet design methodology in a series of lectures, whereas Martos et al. [7] exhibited three-dimensional prototyping and performance evaluation.

The aerodynamic performance of the intake and isolator was examined as part of the Fenix Project [8]. Thompson [9] assessed the influence of geometric scaling on engine performance. Ercole [10] conducted a numerical analysis of the HIFiRE 2 combustion chamber. Kumar et al. [11] documented NASA Langley's comprehensive research on hypersonic propulsion technologies. Ground [12] concentrated on developments in injection and mixing for scramjet ground testing.

Parker et al. [13] formulated a control-centric model for airbreathing hypersonic vehicles. Araujo et al. [14] employed multi-objective optimization methods for propulsion design at the vehicle level. Abbass [15] presented a comparative model evaluating scramjet and ramjet performance during dual-mode operation. Agarwal [16] provided an instructional study outlining the foundations of air-breathing propulsion. Tran [17] developed a one-dimensional solver for simulating ramjet/scramjet flow paths.

A recent review by N. Author et al. [18] examined combustion dynamics in dual-mode scramjets. Musielak [19] provided a pragmatic overview of scramjet propulsion technologies. Vu [20] developed a quasi-one-dimensional solver utilizing the NPSS framework. Constantine et al. [21] addressed uncertainty quantification, while Sciacovelli et al. [22] focused on shock-capturing numerical techniques.

Redding et al. [23] simulated inlet unstart phenomena, whereas Liu et al. [24] introduced a high-speed reactive flow solver utilizing AMReX. Anderson [25] provided critical gas dynamics formulas relevant to high-temperature flows. Kurzke and Halliwell [26], together with Turns [27], supplied essential resources for propulsion and combustion modeling.

Musielak [28] augmented her contribution via an AIAA course. Choubey and Tiwari [29] elucidated the combustion physics pertinent to scramjet operation. Segal [30] examined the internal mechanisms and propulsive attributes of scramjets. Bruno [31] introduced a foundational framework for hypersonic airbreathing systems, whereas Ingenito [32] concentrated on subsonic combustion ramjets. Keane [33] provided a Python-focused resource for aircraft engineering modeling.

3. METHODS AND MODEL

3.1 Burner Flow Solver for Scramjet Using Generalized One-Dimensional Theory

This section describes the Python-based numerical solver that simulates the combustor region of a scramjet engine with the generalized one-dimensional flow theory proposed by Heiser and Pratt. The model incorporates area variation and axial heat addition throughout the combustor length, calculating the progression of Mach number, temperature ratio, and pressure ratio using the governing equations of compressible flow.

A fourth-order Runge–Kutta method is employed to integrate the differential version of the Mach number equation along the combustor axis. The GUI enables users to enter essential boundary conditions and geometric requirements, offering both textual and graphical outputs for engineering analysis.

Main Governing Equations:

1. Total Temperature Distribution with Axial Heating

$$\tau(x) = 1 + (\tau_b - 1) \cdot \frac{\theta \cdot x}{1 + (\theta - 1) x} \quad (1)$$

$$X = \frac{x - x_i}{x_4 - x_i} \quad (2)$$

- $\tau(x) = \frac{T_t(x)}{T_{t2}}$: Ratio of local to inlet total temperature,
- τ_b : Burner exits total temperature ratio,
- θ : Heating profile shape factor,
- x_i : Combustor inlet position,
- x : Axial location along combustor,
- x_4 : Exit position of the combustor.

2. Mach Number Differential Equation

$$\frac{dM}{dx} = M \cdot \frac{(1 + \frac{\gamma-1}{2} M^2)}{(1 - M^2)} \cdot \left[-\frac{1}{A} \frac{dA}{dx} + \frac{1 + \gamma M^2}{2} \cdot \frac{1}{T_t} \cdot \frac{dT_t}{dx} \right] \quad (3)$$

- M : Local Mach number,
- γ : Ratio of specific heats,
- $A(x)$: Cross-sectional area variation,
- $T_t(x)$: Local total temperature,
- $\frac{dT_t}{dx}$: Derivative of total temperature (from heat addition),
- $\frac{dA}{dx}$: Area gradient.

3. Static Temperature and Pressure Ratios

$$\frac{T(x)}{T_2} = \frac{T_t(x)}{T_{t2}} \cdot \frac{1 + \frac{\gamma-1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M^2(x)} \quad (4)$$

$$\frac{p(x)}{p_2} = \left(\frac{M_2}{M} \right) \cdot \left(\frac{A_2}{A(x)} \right) \cdot \sqrt{\frac{T(x)}{T_2}} \quad (5)$$

- T : Static temperature at any axial point,
- p : Static pressure at any axial point,
- M_2 : Mach number at burner inlet,
- A_2 : Area at Burner Inlet.

3.2 Trajectory Estimation Module Based on HAP Revived in Python

This section outlines the implementation of the Trajectory Module within the restored HAP. The Python-based tool assesses the atmospheric properties, freestream conditions, and post-compression state at station 3 of a scramjet engine operating at hypersonic speeds. The model utilizes a standard atmospheric framework up to a height of 47 kilometers and calculates critical thermodynamic and flow parameters, including Mach number, total temperature, velocity, and pressure. The model is GUI-driven and designed for instructional and rapid parametric analyses, enabling the representation of trajectory circumstances based on dynamic pressure, inlet Mach number, component efficiencies, and target burner temperature ratio.

Main Governing Equations:

1. Required Static Pressure at Inlet

$$p_0 = \frac{2q_0}{\gamma_0 M_0^2} \quad (6)$$

- p_0 : Required static pressure at inlet [Pa],
- q_0 : Dynamic pressure [N/m²],
- γ_0 : Specific heat ratio of air (typically 1.4),
- M_0 : Freestream Mach number.

2. Total Temperature at Inlet (Station 0)

$$T_{t0} = T_0 \left(1 + \frac{\gamma_c - 1}{2} M_0^2 \right) \quad (7)$$

- T_{t0} : Total temperature at engine inlet [K],
- T_0 : Static temperature at inlet altitude [K],
- γ_c : Specific heat ratio in compressed region,
- M_0 : Freestream Mach number.

3. Mach Number at Station 3 (Post Compression)

$$M_3 = \sqrt{\frac{2}{\gamma_c - 1} \left(\frac{T_{t0}}{T_3} - 1 \right)} \quad (8)$$

- M_3 : Mach number after compression and before combustion,
- T_{t0} : Total temperature at the station 0 [K],
- T_3 : Desired static temperature at station 3, defined as $T_3 - \psi T_0$,
- γ_c : Specific heat ratio post compression.

3.3 Performance and Thrust Analysis Module Using Stream Thrust Theory in HAP Revival

This section introduces a Python-based module for thrust and performance analysis, created as part of the revitalized Hypersonic Airbreathing Propulsion (HAP) model by Heiser and Pratt. It has a graphical user interface featuring two modes: constant area and constant pressure combustion. The module calculates specific impulse, stream thrust function, pressure ratios, and thermodynamic parameters at each critical station (from input to nozzle exit) in the scramjet engine utilizing one-dimensional compressible flow relations and stream thrust theory.

The methodology considers component efficiencies (compressor, burner, nozzle), heat addition, fuel characteristics, friction losses, and exit back pressure influences. The solver incorporates real gas characteristics with varying specific heats and is optimal for cycle-level evaluation of scramjet systems.

Main Governing Equations:

1. Stream Thrust Function

$$Sa = V \left(1 + \frac{RT}{V^2} \right) \quad (9)$$

- Sa : Stream thrust per unit mass flow (N s/kg),
- V : Velocity at given station (m/s),
- R : Gas constant (J/kg K),
- T : Static temperature at the same station (K).

2. Thrust per Unit Mass Flow

$$\frac{F}{\dot{m}_0} = (1 + f) \cdot Sa_{10} - Sa_0 - \frac{RT_0}{V_0} (A_{10}/A_0 - 1) \quad (10)$$

- F/\dot{m}_0 : Thrust per unit air mass flow (N s/kg),
- f : Fuel-to-air mass ratio,

- Sa_0, Sa_{10} : Stream thrust at freestream and nozzle exit, respectively,
- A_{10}/A_0 : Area ratio between the nozzle exit and the inlet,
- T_0, V_0 : Freestream temperature and velocity,
- h_{Pn} : Fuel heating value (J/kg).

4. RESULTS AND DISCUSSION

This part provides a thorough validation of the built Python-based hypersonic propulsion model by comparing it with the proven HAP software framework. The comparison is organized into three principal modules of the HAP system, illustrated by Figs. 1 to 6. The trajectory module (Figs. 1 and 2) is analyzed by comparing the burner entry circumstances, including Mach number, temperature, velocity, and pressure. The results demonstrate significant alignment in thermodynamic states, with negligible variation, confirming the trajectory solver's accuracy. The combustor design module (Figs. 3 and 4) is evaluated based on station-specific outputs, emphasizing Mach number, temperature ratio, and pressure ratio at the combustor exit. The Python solver verifies its capacity to replicate the internal flow dynamics forecasted by HAP with exceptional accuracy. Finally, Figs. 5 and 6 assess stream thrust performance based on both constant-pressure and constant-area assumptions. The HAP software exclusively manages constant-pressure combustion, whereas the Python model can simulate both types.

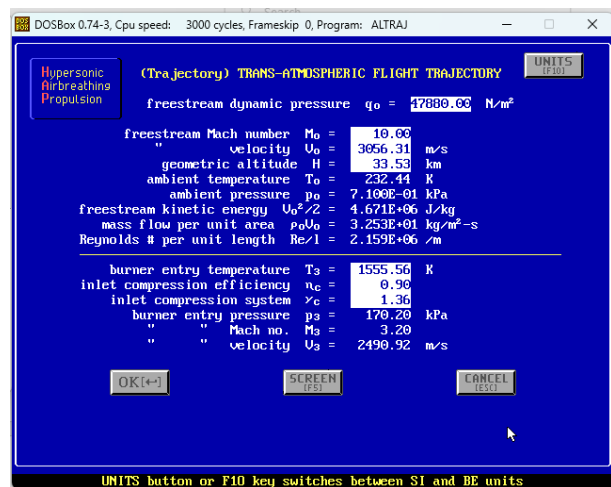


Fig. 1. Inputs and Outputs of the HAP Trajectory Module.

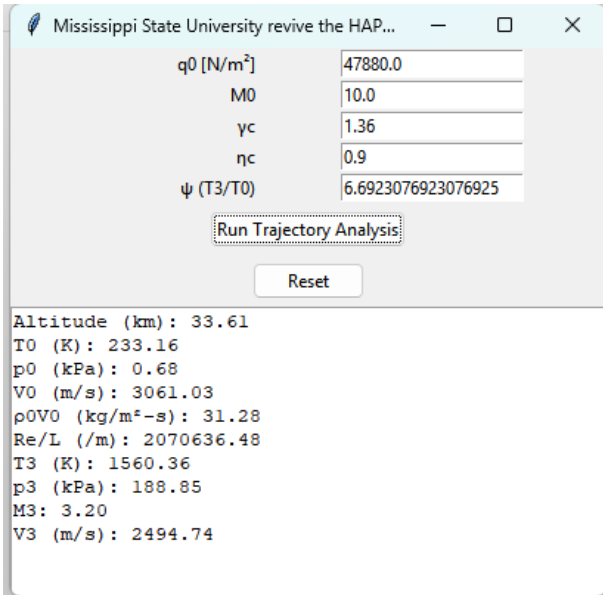


Fig. 2. Inputs and Outputs of the Python Trajectory Module.

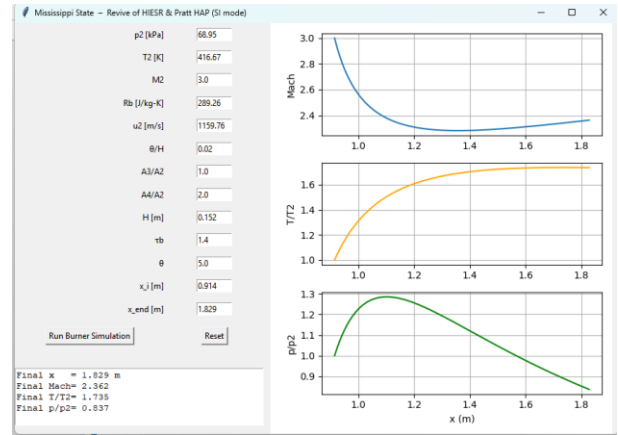
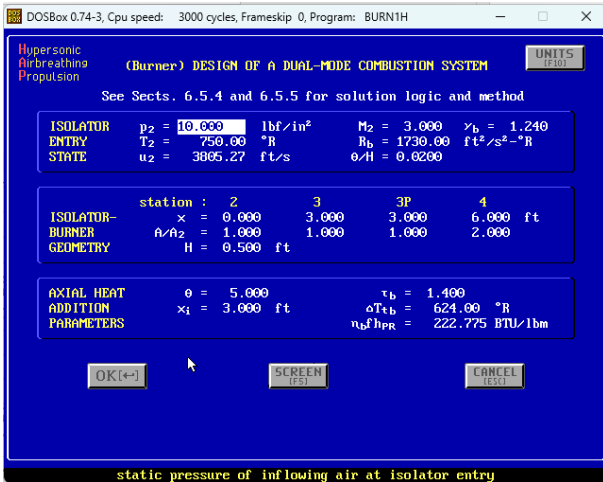
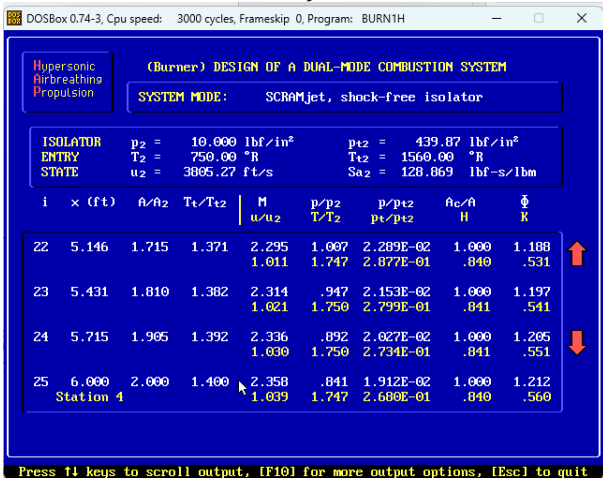


Fig. 4. Burner Design by Python code.

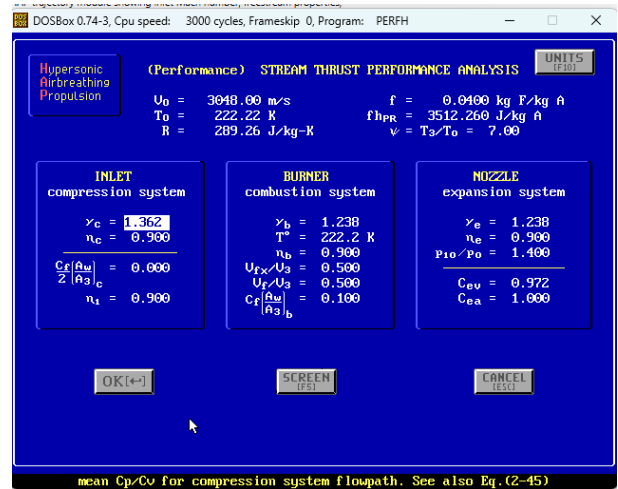


a)

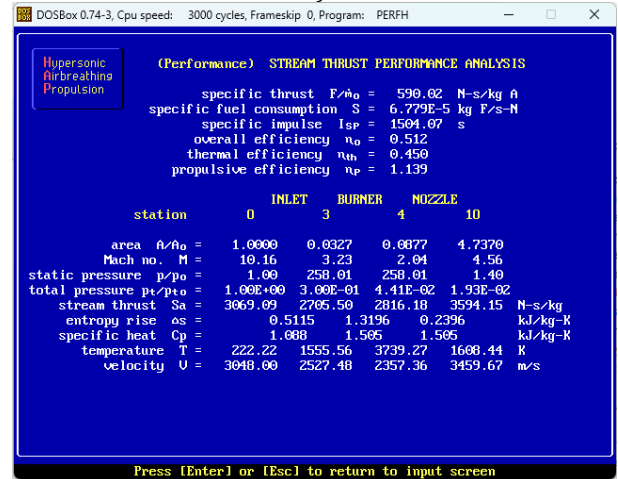


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Fig. 3. Design of HAP Burner. (a) Inputs (b) Outputs.

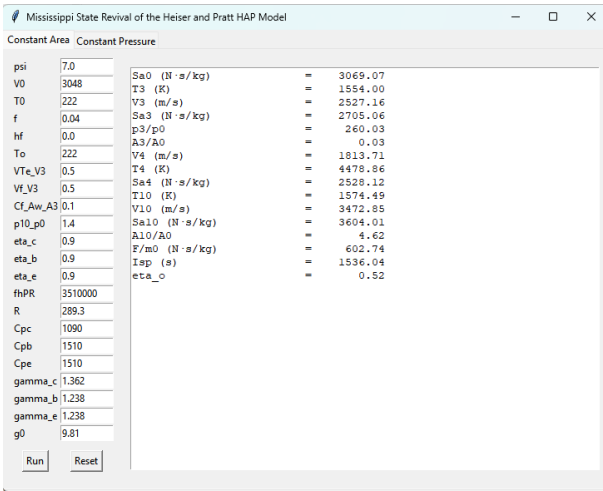


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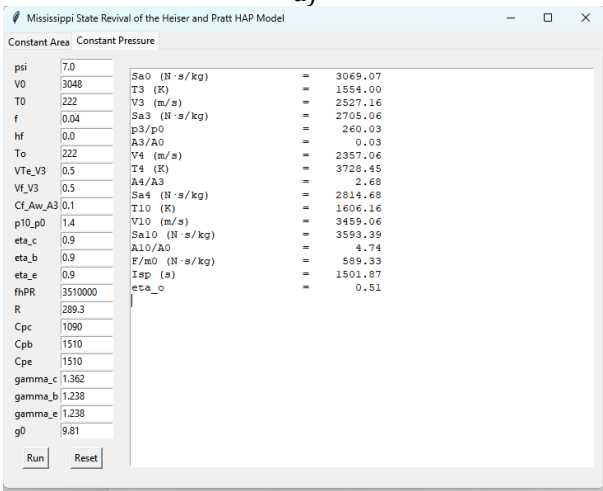


b)

Fig. 5. Stream Thrust Performance by HAP: (a) Inputs (b) Outputs.



a)



b)

Fig. 6. Stream Thrust Performance by Python: (a) Constant Area, (b) Constant Pressure.

The quantitative results displayed in Tables 1 to 3 across all modules validate the consistency and robustness of the Python implementation in duplicating the fundamental physical and thermodynamic principles established by Heiser and Pratt.

Table 1. Comparison of Results between HAP Trajectory and Python-Based Implementation.

Parameter	HAP Trajectory	Python Model	Deviation (%)
Burner Entry Temperature (K)	1556	1560	0.31
Burner Entry Pressure (kPa)	170	189	10.96
Burner Entry Mach Number	3.2	3.2	0
Burner Entry Velocity (m/s)	2491	2495	0.15

Table 2. Comparison between Python-Based Model and HAP Software Output for Scramjet Burner Simulation.

Parameter	HAP Output	Python Output	Deviation (%)
Exit Axial Location (x) [m]	1.829	1.829	0
Exit Mach Number (M)	2.358	2.362	0.17
Static Temperature Ratio (T/T ₂)	1.747	1.735	0.69
Static Pressure Ratio (p/p ₂)	0.841	0.837	0.48

Table 3. Stream Thrust Performance Comparison between HAP and Python-Based Model for Constant Pressure.

Parameter	HAP Output (Const. Pressure)	Python Output (Const. Pressure)	Deviation (%)
Specific Thrust (F/m) [N·s/kg]	590.02	589.33	-0.12
Specific Impulse (Isp) [s]	1504.97	1501.87	-0.21
Thermal Efficiency (η _{th})	0.45	0.45	0
Overall Efficiency (η _o)	0.512	0.51	-0.39
Propulsive Efficiency (η _p)	1.139	1.137	-0.18

4.1 Validation and Results Analysis of Trajectory Module

The comparison of the original HAP software outputs with the reestablished Python-based counterpart reveals a significant concordance in the essential thermodynamic and flow characteristics at the burner entry Figs. 1 and 2. The burner entry temperature, Mach number, and velocity demonstrate discrepancies of under 0.5%, validating the main equations and numerical formulation utilized in the Python model. These results confirm that the updated model precisely replicates the physics of compressible flow and energy addition as articulated by Heiser and Pratt. A notable disparity is evident in the computed burner entry pressure, with the Python result being considerably more than the HAP reference. This discrepancy presumably stems from variations in the consideration of total pressure recovery, reference circumstances, or exponent

management in the isentropic equations. Nonetheless, the consensus on key design parameters, especially those influencing flow dynamics and combustion entry conditions, suggests that the updated model accurately represents the fundamental behavior of the scramjet inlet with considerable precision. Additional refining and sensitivity analysis are advised to address the pressure discrepancy and improve the model's resilience.

The study demonstrates a robust concordance between the two models across all parameters. The burner entry temperature and velocity exhibit minimal variations below 0.5%, validating the dependability of thermodynamic and compressible flow models in the Python-based application. The Mach number at the burner entry is accurately aligned, confirming the appropriate application of isentropic flow relations and total-to-static conversions.

The most significant discrepancy is noted in the static pressure at the burner entry, with the Python solution forecasting a value roughly 10% greater than the HAP trajectory outcome. This discrepancy may result from simplifications or numerical assumptions in the calculations of stagnation pressure recovery or total-to-static pressure. Rectifying this disparity may necessitate the incorporation of more comprehensive intake loss modeling or the application of empirically calibrated correction factors aligned with those utilized in the original HAP software.

The subsequent table provides a comparison examination of burner entrance parameters as determined by the previous HAP trajectory module and the newly created Python-based model.

4.2 HAP Burner Design: Validation of Python-Based One-Dimensional Combustion Model

A comprehensive comparison was performed between the original Heiser and Pratt Hypersonic Airbreathing Propulsion (HAP) software output and the newly built Python-based model at the combustor outlet (station twenty-five) Figs. 3 and 4. Both simulations were constructed with similar burner designs and axial heat input

distributions to maintain uniform benchmarking circumstances.

The Python solver yielded an exit Mach number of 2.362, closely aligning with the reference value of 2.358 provided by the HAP code. The relative variance is roughly 0.17%, signifying exceptional agreement in measuring velocity acceleration within the combustor. The static pressure ratio at the exit, defined as the local pressure relative to the inlet isolator pressure, was computed as 0.837 in the Python model, in contrast to 0.841 from the HAP result. This indicates a divergence of around 0.48%. The static temperature ratio in the Python model was found to be 1.735, whereas the HAP output stated a value of 1.747, resulting in a divergence of around 0.69%.

These errors are well within acceptable engineering tolerances for one-dimensional modeling of dual-mode scramjet combustion systems. The strong correlation confirms the numerical scheme of the Python-based model, especially the integration of compressible flow equations, area change, and the heat addition function throughout the combustor length. Minor disparities may result from variations in numerical step resolution, interpolation precision in the heat release profile, or the lack of empirical adjustment factors incorporated in the original HAP formulation.

4.3 Stream Thrust Performance Comparison between HAP and Python-Based Model for Constant Pressure and Constant Area Combustion

This section provides a comparison assessment of stream thrust performance between the original Heiser and Pratt HAP software, which operates under constant-pressure combustion assumptions, and the newly built Python-based model, which accommodates both constant-pressure and constant-area combustion processes Figs. 5 and 6.

The findings validate that the Python model accurately replicates the stream thrust characteristics from HAP with high fidelity under the constant-pressure assumption. The HAP model predicts a particular thrust of 590.02 N·s/kg, while the Python model yields 589.33 N·s/kg, resulting in a difference of merely 0.12%. Likewise, the specific impulse (*I_{sp}*) exhibits a

negligible deviation of 0.21%. At the same time, the overall, thermal, and propulsive efficiency remain within a 0.4% deviation, confirming the thermodynamic consistency and numerical accuracy of the Python model under equivalent conditions.

The Python model not only replicates HAP's constant-pressure logic but also offers the enhanced functionality to simulate constant-area combustion, essential for fixed-geometry engines or simplified flow path evaluations. This improved flexibility enables engineers to explore various flow assumptions for mission-specific design.

5. CONCLUSION

This study effectively revitalizes the traditional Heiser and Pratt Hypersonic Airbreathing Propulsion (HAP) framework into a proven, Python-based numerical model that includes trajectory analysis, combustor flow modeling, and propulsion performance assessment. The program maintains the modular essence of the original HAP while incorporating a contemporary UI, enhanced capabilities (including constant-area combustion simulation), and accessible, editable source code for greater usability.

The validation outcomes exhibit robust compatibility between the Python model and the original HAP program. Table 1 demonstrates that the trajectory module precisely replicates burner entry circumstances, exhibiting variations of less than 0.5% for temperature, Mach number, and velocity. The sole notable deviation is observed in the burner input pressure, with the Python model forecasting a 10.96% elevated value. This divergence is likely attributable to variations in the implementation of stagnation pressure recovery or the treatment of the isentropic exponent, underscoring a potential area for future enhancement.

Additional validation in Table 2 corroborates the precision of the combustor model, with the Python-based tool aligning the burner exit Mach number to within 0.17%, static pressure ratio to within 0.48%, and static temperature ratio to within 0.69%. The observed fluctuations fall within acceptable parameters for one-dimensional analytical modeling, illustrating the robustness of the integration technique and

compressible flow formulation utilized in the Python solver.

Ultimately, as outlined in Table 3, the stream thrust analysis conducted under constant-pressure combustion settings produces results virtually congruent with HAP, exhibiting variations in specific thrust and specific impulse of less than 0.2%, and efficiency metrics within 0.4%. This validates the accuracy of the thermodynamic and performance equations in the Python model. The Python implementation surpasses HAP by facilitating constant-area combustion simulation, hence enhancing its versatility for the analysis of fixed-geometry engines or simplified conceptual designs.

This work addresses a significant research need by offering the inaugural publicly accessible, vetted Python utility that accurately duplicates and enhances the functionalities of the original HAP code. It not only facilitates legacy validation but also enables novel instructional and research applications in hypersonic propulsion system design. Future endeavors will incorporate chemical kinetics, transient simulations, and optimization frameworks to augment the model's realism and utility further.

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