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A Numerical Model for Scramjet and Ramjet Efficiency Study during Dual and Individual Modes of Operation

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A B S T R A C T

Hypersonic airbreathing propulsion, specifically scramjet technology, represents a transformative advancement in high-speed flight. This literature review examines critical studies and technological developments that have contributed to our understanding and implementation of scramjet systems. The primary reference is the comprehensive resource provided by Utah State University, which offers detailed insights into various propulsion systems. Acknowledgments are due to Dora E. Musielak, for her contributions through the AIAA Training Course on Hypersonic Air Breathing Propulsion in 2018. Foundational efforts by Heiser and Pratt (1995) laid the groundwork for theoretical and practical aspects of hypersonic propulsion. Significant advancements include the development and testing of the X-43A vehicle, achieving Mach 10, and subsequent studies addressing materials, thermal protection, and aerodynamic heating challenges. Further research has optimized scramjet performance through advanced design techniques and computational simulations. This review highlights contributions from notable works on scramjet inlets, combustion systems, propulsion system airframe integration, and numerical simulations of thermodynamic non-equilibrium. The innovative design of the Rectangular-to-Elliptical Shape Transition (REST) scramjet inlet/engine is also discussed, showcasing the progress in hypersonic propulsion technology. In addition to reviewing the literature, this study presents a Python code developed for modeling scramjets or ramjets with options for normal shocks or combined shocks. The code allows users to study one of three models and analyze efficiency against inlet Mach number or turning angles. It also demonstrates the difference in performance under dual operation ramjet/scramjet operations, providing a practical tool for evaluating and comparing propulsion system efficiencies.

1. INTRODUCTION

The research into hypersonic airbreathing propulsion, namely scramjet technology, has received considerable attention due to its capacity to transform high-speed flying. This literature review focuses on important studies and breakthroughs in this topic, specifically highlighting the theoretical and practical progress that has influenced present knowledge and abilities. The main source for this review is the exhaustive resource offered by Utah State University, which extensively covers several facets of propulsion systems [1]. The acknowledgement should be credited to Dora E. Musielak, for her contribution to the AIAA Training Course on Hypersonic Air Breathing Propulsion in 2018 [2]. The pioneering work of Heiser and Pratt [3] offers a fundamental examination of hypersonic airbreathing propulsion systems [3], specifically focusing on the principles and difficulties related to scramjet engines. Their contributions to the AIAA Education Series continue to serve as an essential resource for comprehending the theoretical foundations of hypersonic propulsion. Multiple studies provide comprehensive documentation on the construction of the X-43A vehicle, which successfully attained a speed of Mach 10. The study conducted by Ohlhorst et al. [4] specifically examines the leading edges of objects moving at Mach 10. The researchers investigate the various materials and thermal protection obstacles that are encountered in this context. Marshall et al. [5] provide a chief engineer's viewpoint on the NASA X-43A scramjet flight test, focusing on the technical challenges encountered and the corresponding solutions developed during these groundbreaking experiments. In addition, Berry et al. [6] investigate the transition of the boundary layer on the X-43A, yielding useful information on the aerodynamic heating and stability implications for hypersonic vehicles. Ferguson and Dhanasar [7] investigate additional progress in scramjet technology by proposing a paradigm for the design and evaluation of thrust-optimized scramjets. Their work highlights the significance of enhancing performance using sophisticated design strategies. In Smart's [8] study, the author explores different designs for scramjet inlets and examines how these configurations affect the efficiency of the engine. Fulton et al. [9] employ large-eddy simulations to simulate reactive flow in a dual-mode scramjet combustor, specifically studying the intricate interplay between combustion and flow at hypersonic velocities. Huebner and Tatum [10] conducted a study where they calibrated CFD programs and investigated the impact of inlet fairing on a 3D hypersonic powered-simulated model. Their work improved the accuracy of hypersonic flow simulations by boosting prediction capabilities. Neill and Pesyridis [11] address the task of modelling supersonic combustion systems for prolonged hypersonic flight. They emphasize the essential components required to ensure stable combustion in high-speed conditions. Mutzman and Murphy [12] present an analysis of the X-51 scramjet's development from the viewpoint of a chief engineer. They discuss the various engineering obstacles faced and the corresponding solutions implemented to accomplish continuous hypersonic flight. Other significant works that have had a strong impact include Kerrebrock's [13] comprehensive examination of aviation engines and gas turbines, and Mattingly and Boyer [14] essential reference on gas turbines and rockets for propulsion systems. Engelund et al. [15] examine the challenges of integrating propulsion systems with airframes and the construction of aerodynamic databases for the Hyper-X flying research vehicle. Meanwhile, Lee et al. [16] explores innovative technologies for high-speed propulsion. Vân Wie et al. [17] offer comprehensive analysis on the subject of scramjet inlets and supersonic combustion ramjet systems, respectively. Glass et al. [18] investigates the science and technology related to the structure of hypersonic airbreathing vehicles. Marshall et al. [19] provide the perspective of a lead engineer on the NASA X-43A scramjet flight test. Erbland et al. [20] provide more analysis of the integration and operational elements of scramjet vehicles, focusing on near-term initiatives for the development of hypersonic vehicles. Fiévet et al. [21] examine the numerical simulation of a scramjet isolator with thermodynamic nonequilibrium, offering important insights into the functioning of scramjet systems under different circumstances. The Rectangular-to-Elliptical Shape Transition (REST) scramjet inlet/engine, as described by NASA in 2002, demonstrates the need for creative design methods to achieve efficient hypersonic propulsion.

2. APPROACH AND SPECIFICS OF THE RESEARCH

2.1 An overview of Hypersonic Vehicles

Hypersonic vehicles attain velocities surpassing Mach 5, which is almost fivefold the speed of sound. The high velocities pose significant challenges in terms of aerodynamic and thermal forces, which need to be carefully addressed in the design process. Hypersonic vehicles attain velocities surpassing Mach 5, which is almost fivefold the speed of sound. The high velocities pose significant challenges in terms of aerodynamic and thermal forces, which need to be carefully addressed in the design process.

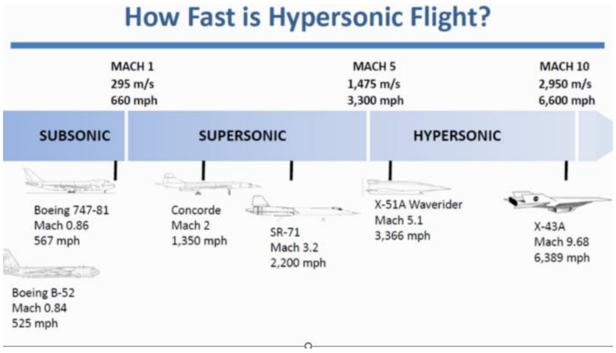

Fig 1. Hypersonic Flight compared to other flights [15].

Hypersonic speed refers to speeds that are extremely fast, typically exceeding five times the speed of sound. Hypersonic speed is a flow regime where the transfer of energy between the flow and thermodynamic and chemical processes becomes considerable. In regimes where Mach numbers above 5, conventional supersonic theories become inadequate due to the non-constant specific heat ratio and the need to account for temperature effects on fluid properties. The physics of flight at these speeds is mostly governed by aerodynamic heating.

2.2 Challenges in designing scramjets

Challenges in Designing Inlets

The hypersonic vehicle's intake system collects and compresses the incoming air to provide it to the combustor. Several primary barriers include: When the Mach values exceed 5, the inlet undergoes significant aerodynamic heating. Surfaces must be dulled to reduce heat generation. The occurrence of the "shock-on-lip" scenario is limited to situations where the aircraft is flying at its specified Mach number. The system's performance is adversely impacted when it undergoes movements induced by ramp shock under non-standard conditions. The aircraft's forebody features a blunted leading edge that generates a concentrated layer of entropy, resulting in a more intricate design. Efficient operation requires the optimization of high inlet pressure recovery. Entropy Swallowing is the method of efficiently managing the increase in entropy resulting from shock waves and high-speed flow conditions. It is of utmost importance to guarantee the highest level of performance and stability while designing hypersonic vehicles.

Isolator Design Challenges

The primary function of the isolator is to separate the inlet from any disruptions caused by the discharge of heat in the combustor, while also facilitating the formation of shock trains. Some of the main difficulties are: The Shock Train Formation involves the combination of a powerful shock wave, separation of the boundary layer, and an overexpansion process, which functions as a natural valve. The isolator must have a sufficient length to effectively separate the inlet from any disturbances that may be transmitted from the combustor in the upstream direction.

Combustor Design Challenges

The combustor combines and ignites fuel with air. Some of the main difficulties are. Supersonic Efficiently combining fuel and air in a short amount of time is crucial for optimal mixing. Heat Release involves the control of heat release to prevent thermal constriction and excessive pressure differences. Fire it is essential to have devices that can securely hold the flame and maintain a consistent combustion.

Challenges in Designing Nozzles

The nozzle increases the speed of the flow in order to generate propulsion. Some of the main difficulties are. The nozzle geometry needs to be tuned to achieve the highest possible thrust, while considering how the exhaust interacts with the surrounding flow. Three-dimensional expansion To achieve maximum thrust, a 3D compression inlet necessitates 3D expansion in the nozzle.

Computational Fluid Dynamics (CFD) is effective

Computational Fluid Dynamics (CFD) is essential for the simulation and examination of high-velocity fluid flows. Important study points encompass the modeling of shock layers, which are small layers of compressed gas that hamper the construction of vehicles. The study focuses on the impact of increasing entropy caused by the strength of shocks on different layers. Analyzing the relationship between the growing flow temperature and the thickness of the boundary layer in a viscous interaction. The study focuses on investigating the effects of high temperatures on aerodynamic heating and how it affects the performance of vehicles. Low density flow simulates situations that challenge typical aerodynamic model principles.

Types of Fuel

Hydrogen-powered cars need to have a high volumetric efficiency in order to store a fuel with low density. Hydrocarbon-powered vehicles prioritize loading efficiency since the fuel has a high density. Ratio of air to fuel Optimizing the air-to-fuel ratio is essential for achieving efficient combustion. Effective control of this ratio guarantees peak performance and fuel economy.

2.3 Types of Analysis

Thermodynamic Cycle Analysis

Thermodynamic cycle analysis involves studying engine reference stations and their properties. Key parameters include:

- Flow Thermodynamic Properties,
- Maximum Allowable Compression Temperature,
- Required Burner Entry Mach Number,
- Fuel/Air Ratio and Equivalence Ratio,
- Combustor Energy Balance,
- Combustor Total Pressure Loss.

Reference Station	Engine Location
0	Undisturbed or freestream condition External compression begins
1	External compression ends Internal compression begins Inlet or diffuser entry
3	Inlet or diffuser exit Internal compression ends Combustor entry
4	Combustor exit Internal expansion begins Nozzle entry
9	Internal expansion ends Nozzle exit External expansion begins
10	External expansion ends

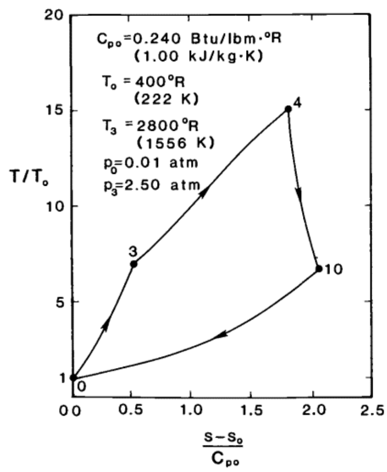

Fig 2. Thermodynamic cycle [2].

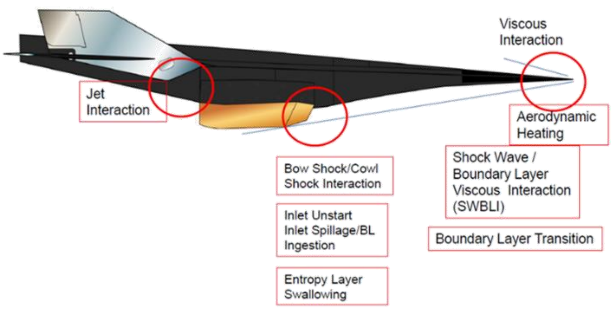

Fig 3. Challenges of scramjet [15].

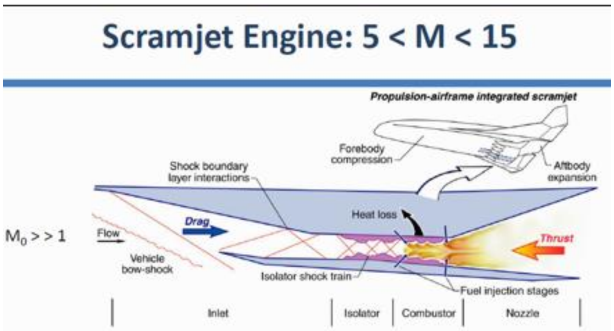

Fig 4. Scramjet flow path [15].

Thrust Analysis

Thrust is produced by accelerating the flow in the nozzle. Key factors influencing thrust include:

- Inlet and Combustor Performance,
- Nozzle Geometry,
- Flight Mach Number,
- Dynamic Pressure.

2.4 Performance Measures

Key performance measures for hypersonic vehicles include Specific Thrust, Specific Impulse, and Propulsive and Overall Efficiencies.

Difference between Ramjet and Scramjet:

- Ramjet: Operates efficiently at subsonic combustion speeds (Mach 3-6),
- Scramjet: Operates efficiently at supersonic combustion speeds (Mach 5-15).

2.5 Mathematical modelling of Scramjet/Ramjet

- Model concept 1: Dimensional Enthalpy to Dimensional Kinetic Energy. This Model concept requires inputs such as fluid density, velocity, and specific heat capacity, and it outputs the relationship between enthalpy and kinetic energy. Understanding this relationship is crucial for energy distribution in hypersonic flows, which impacts thermal management and propulsion efficiency.
- Model concept 2: Dynamic Pressure. This Model concept takes fluid density and velocity as inputs and outputs the dynamic pressure. Quantifying the pressure exerted by the fluid flow is crucial for designing components like inlets and isolators to withstand high-pressure environments.
- Model concept 3: Isolator Length. Inputs for this Model concept include shock wave properties and duct length, and it outputs the required length of the isolator. Determining the appropriate length is critical for ensuring the isolator's effectiveness and maintaining stable combustion conditions.
- Model concept 4: HAP Vehicle Range & Main Performance Parameters. This Model concept requires vehicle properties and flight conditions as inputs and outputs the range and performance metrics. It provides key performance indicators for hypersonic vehicles, aiding in the design of propulsion systems and overall vehicle efficiency.
- Model concept 5: Calorifically or Thermally Perfect Cp Function in Temperature. The inputs for this Model concept are temperature and specific heat capacity, and it outputs the relationship between temperature and specific heat capacity. This relationship is essential for modelling thermal properties of air at high temperatures, influencing thermal management strategies in hypersonic vehicles.
- Model concept 6: Maximum Allowable Temperature & Mach Number for Burner. This Model concept requires temperature limits and Mach number as inputs, and it outputs the allowable temperature and Mach number for safe operation. Ensuring the burner operates within safe temperature limits is crucial for preventing material degradation and ensuring efficient combustion.
- Model concept 7: Performance Measures for HAP. Inputs for this Model concept include various flight and vehicle parameters, and it outputs performance measures such as specific thrust and impulse. Quantifying performance measures helps in evaluating the effectiveness of propulsion systems and optimizing vehicle design for different flight regimes.
- Model concept 8: Viscosity Formula. This Model concept requires temperature and viscosity coefficients as inputs and outputs the viscosity of the fluid. Accurate viscosity calculations are essential for predicting flow behaviour and designing components to handle viscous effects.
- Model concept 9: Entropy Swallowing. Inputs for this Model concept include pressure and temperature changes, and it outputs entropy changes. Managing entropy increases is crucial for maintaining high efficiency and avoiding performance losses due to thermal effects.
- Model concept 10: Inlet Performance Parameters. This Model concept requires inlet geometry and flow conditions as inputs and outputs performance parameters such as pressure recovery and efficiency. Optimizing inlet performance parameters is essential for ensuring adequate air mass flow and minimizing losses.

- Model concept 11: Kantrowitz Limit for Contraction. Inputs for this Model concept include inlet area ratios and Mach number, and it outputs the maximum allowable contraction ratio. Ensuring the inlet design adheres to the Kantrowitz limit is crucial for preventing flow separation and maintaining high efficiency.
- Model concept 12: Sizing Isolator. This Model concept requires shock properties and isolator length as inputs and outputs the dimensions of the isolator. Properly sizing the isolator ensures it can manage shock trains effectively, crucial for stable engine operation.
- Model concept 13: Sizing Nozzle. Inputs for this Model concept include nozzle geometry and flow properties, and it outputs the dimensions of the nozzle. Accurate nozzle sizing is essential for maximizing thrust and ensuring efficient exhaust expansion.
- Model concept 14: Heat Addition. This Model concept requires fuel properties and combustion parameters as inputs and outputs the heat added to the system. Understanding heat addition is vital for managing thermal loads and ensuring efficient combustion.
- Model concept 15: Combustion Process. Inputs for this Model concept include fuel properties and air properties, and it outputs combustion efficiency and energy release. Detailed modelling of the combustion process ensures efficient energy conversion and minimizes losses.
- Model concept 16: Dissociation Effect on Combustion. This Model concept requires temperature, pressure, and chemical species as inputs and outputs the degree of dissociation. Accounting for dissociation effects is crucial for accurate modelling of high-temperature combustion processes.
- Model concept 17: Combustion Burning Time. This Model concept requires combustor length and flow velocity as inputs and outputs the burning time. Ensuring adequate burning time is essential for maximizing combustion efficiency and energy release.
- Model concept 18: Fuel Penetration Model concept. Inputs for this Model concept include fuel properties, air properties, and flow velocity, and it outputs the penetration depth. Proper fuel penetration is critical for achieving optimal mixing and combustion in the combustor.
- Model concept 19: Combustion One-Dimensional Model concept. This Model concept requires combustor dimensions and flow properties as inputs and outputs combustion characteristics. One-dimensional analysis provides a baseline understanding of the combustion process, aiding in initial design and optimization.
- Model concept 20: Drag, Lift, & Friction. Inputs for this Model concept include aerodynamic properties and vehicle.
- Model concept 21: Aerodynamic Factors. This Model concept requires flow properties and vehicle geometry as inputs and outputs aerodynamic performance metrics. Managing aerodynamic factors is essential for maintaining high efficiency and minimizing performance losses.
- Model concept 22: Aerodynamic Reference Quantities. This Model concept requires flow properties and vehicle dimensions as inputs and outputs reference quantities. These quantities help in understanding and predicting the behaviour of hypersonic flows, informing design decisions.
- Model concept 23: Aerodynamic Heating. This Model concept requires flow velocity and vehicle surface properties as inputs and outputs the heat transfer rate. Effective thermal management strategies are vital for preventing structural damage and ensuring long-term reliability.

2.6 The modelling of ScramJet and Ramjet first law of thermodynamic analysis with compressible flow models across the flow path

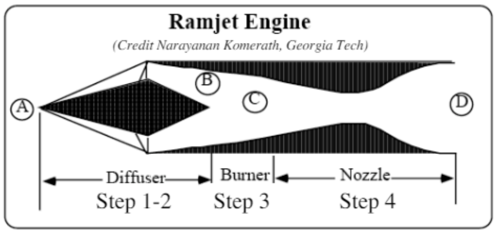

Fig 5. Ramjet Schematic [1].

- The ratio of engine pressure, P-B/P-A, grows accordingly The magnitude of η is growing.[1]

- The temperature difference between the combustor, *T-C.–*, and the burner, *T-B.*, grows, the efficiency, η, also increases [1].
- The ratio of the total pressure at the intake (P_0_B) to the total pressure at the outlet (P_0_A) declines, and the loss of stagnation pressure increases, leading to a decrease in efficiency (η).[1]
- Thermal Efficiency Equation[1]:

$$\eta = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \frac{\left(T_C - \left(\frac{P_{0_B}}{P_{0_A}}\right)^{\frac{\gamma-1}{\gamma}} T_B\right)}{(T_C - T_B)} \quad (1)$$

- Chocked Nozzle Throat [1]:

$$\frac{\dot{m}}{A^*} = \sqrt{\frac{\gamma}{R_g}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\,\frac{p_0}{\sqrt{T_0}} \quad (2)$$

- Isentropic Flow Model concepts:

$$\frac{T}{T_o} = \left(\frac{\rho}{\rho_o}\right)^{\gamma-1} = \left(\frac{P}{P_o}\right)^{\frac{\gamma-1}{\gamma}} \quad (3)$$

$$\frac{\rho}{\rho_o} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{1}{\gamma-1}} \quad (4)$$

$$\frac{P}{P_o} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma}{\gamma-1}} \quad (5)$$

$$\frac{A}{A_*} = \frac{1}{M}\left(\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right)^{\frac{1\gamma+1}{2\gamma-1}} \quad (6)$$

- Oblique Shock Wave Relations [1]:

$$M_2^2 \sin^2(\beta - \theta) = \frac{1 + \frac{\gamma-1}{2}M_1^2 \sin^2\beta}{\gamma M_1^2 \sin^2\beta - \frac{\gamma-1}{2}} \quad (7)$$

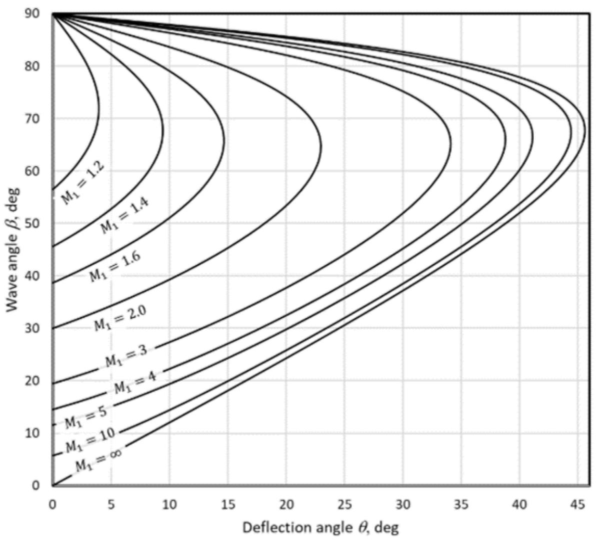

Fig 6. Oblique shock wave.

$$\frac{T}{T^*} = M^2\left(\frac{1+\gamma}{1+\gamma M^2}\right)^2 \quad (8)$$

$$\frac{\rho}{\rho^*} = \frac{1}{M^2}\left(\frac{1+\gamma M^2}{1+\gamma}\right) \quad (9)$$

$$\frac{p_0}{p_0^*} = \frac{1+\gamma}{1+\gamma M^2}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}} \quad (10)$$

$$\frac{T_0}{T_0^*} = \frac{(1+\gamma)M^2}{(1+\gamma M^2)^2}\left[2 + (\gamma-1)M^2\right] \quad (11)$$

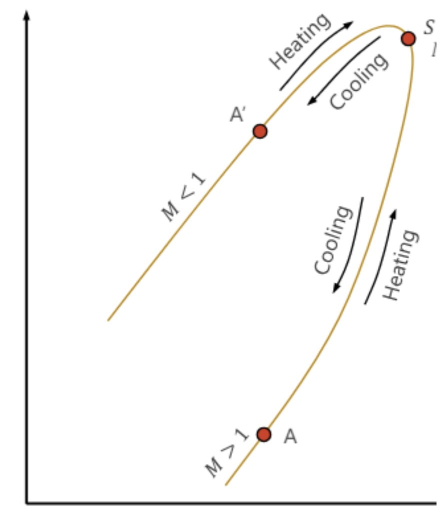

Fig 7. Rayleigh flow.

2.7 Design Cases

Summary of 2-D Ramjet Inlet Example Normal shock wave [1]:

- The inlet Mach number before the normal shock wave is M1 = 4.0.
- After the normal shock wave, the Mach number reduces to M_B = 0.434959.
- The total pressure ratio across the shock wave is P0_B / P0_∞ = 0.1388.
- The static pressure ratio across the shock wave is P_B / P_∞ = 18.5.
- Static temperature after burner (TBC) is 1356.3 K.
- The efficiency calculation involves comparing the temperatures before and after the burner.
- The formula accounts for the changes in total pressure and temperature across the normal shock.
- The efficiency is derived by considering the losses due to the shock and the subsequent temperature changes.

Result: The calculated efficiency is 0.2328.

$$T_{0_c} = \frac{q + c_p T_{0\infty}}{c_p}$$

$$= \frac{500 \cdot 10^3 + 1004.696(909.93)}{1004.696} = 1407.6°\text{K} \quad (12)$$

- The free stream stagnation temperature (T0∞) before the burner is 909.93 K.
- The static temperature before the burner, denoted as TeB, is determined by considering the isentropic relationship between stagnation temperature and static temperature, taking into consideration the Mach number before to entering the burner. The outcome is 867.75 Kelvin.
- After Burner Stagnation Temperature after Burner (T0C): The temperature at stagnation point after the burner is 1407.6 K.
- The static temperature after the burner, also known as the TBC (Temperature Before Combustion), is determined by using the isentropic relationship between stagnation temperature and static temperature, taking into account the Mach number after the burner. The outcome is 1356.3 Kelvin.

Summary of 2-D Ramjet Inlet Example across Oblique Shock Wave then normal shock wave [1]:

- Oblique Shock Angle (β): 40°.
- Mach Number after Oblique Shock (M2): 2.123.
- Mach Number after Normal Shock (MB): 0.557853.
- Total Pressure Ratio across Oblique Shock (P0B / P02): 0.663531.
- Combined Total Pressure Ratio (P0B / P0∞): 0.3126.
- Static Pressure Ratio (PB / P∞): 38.422.
- This result in efficence of 0.4652.
- Or the inlet oblique shock doubled the efficiency.

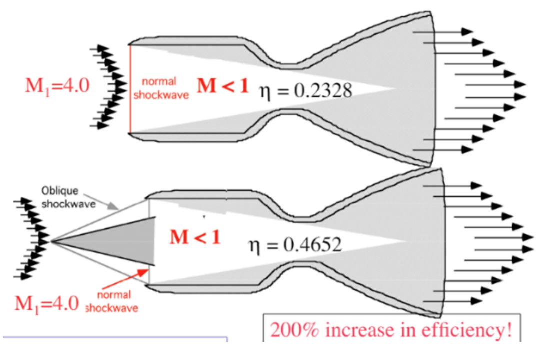

Fig 8. Comparison between Ramjet operation with inlet oblique and normal shock [1].

For a pure Scramjet model with two oblique shock waves at the inlet [1]:

- Initial Mach Number: The inlet starts with a high Mach number (M1 = 3.6).
- First Oblique Shock: An oblique shock forms, reducing the Mach number to 2.3552 and causing a flow deflection.
- Second Oblique Shock: Another oblique shock forms, further reducing the Mach number to 1.5333 and causing additional flow deflection.
- Supersonic Flow: Despite the shocks and deceleration, the flow remains supersonic after the shocks, which is crucial for the Scramjet operation.
- This will produce 6-10% increase in efficiency.

2.8 The Python Code

The given Python code functions as a comprehensive tool for simulating the effectiveness of scramjet or ramjet engines under varied stress circumstances. It enables users to evaluate engine performance by considering different input parameters. The code incorporates functions for computing temperature and pressure ratios, shock characteristics, and efficiency. Users initiate the process by choosing the specific case to be analyzed: Case 1 pertains to a regular shock, Case 2 pertains to an oblique shock followed by a regular shock, and Case 3 pertains to two oblique shocks. Once the case is chosen, users enter the Mach number, which indicates the velocity of the airflow in relation to the speed of sound. In circumstances involving oblique shocks (namely Case 2 and Case 3), users are required to input the turning angle. This angle represents the deflection of the airflow as it passes through the shock wave. Furthermore, users input the intake temperature and inlet pressure, which refer to the temperature and pressure of the air prior to its entry into the engine. The quantity of thermal energy introduced into the engine, which signifies the energy provided to the air-fuel combination in the combustor, is an additional crucial input.

The code utilizes the given inputs to compute the static temperature prior to the burner (TB), the static temperature subsequent to the burner (TC), and the efficiency of the engine. The static temperature prior to the burner is computed based on the chosen shock condition and input parameters, whereas the static temperature following the burner is determined by adding the heat input to the temperature prior to the burner. The efficiency is calculated by considering the temperatures before and after the burner, as well as the pressure ratio across the shocks.

The code consists of multiple essential functions. The `isentropic_temperature_ratio` function computes the ratio of total (stagnation) temperature for isentropic flow, given the Mach number and specific heat ratio. The `normal_shock_temperature_ratio` function calculates the temperature ratio over a normal shock wave based on the inlet Mach number and specific heat ratio. The `oblique_shock_angle` function calculates the shock angle by employing numerical root-finding techniques, based on the provided turning angle and input Mach number. The `oblique_shock_properties` function computes the characteristics of the fluid flow following an oblique shock, such as the Mach number, pressure ratio, and temperature ratio after the shock. The `efficiency` function computes the efficiency of the propulsion system by considering the temperatures prior to and subsequent to the burner, as well as the pressure ratio. The `calculate_efficiency` function computes the efficiency by evaluating the temperature and pressure ratios associated with the chosen scenario and using the efficiency formula.

The function `plot_efficiency_vs_mach` visualizes the relationship between efficiency and Mach number for the specified scenario and input parameters. Similarly, the function `plot_efficiency_vs_turning_angle` visualizes the relationship between efficiency and turning angle. The main function acts as the starting point of the script, where it prompts the user for inputs, calculates the efficiency for the chosen scenario, and optionally plots the efficiency vs Mach number or turning angle. This code is a significant tool for simulating and evaluating the performance of scramjet or ramjet engines. It enables users to obtain a deep understanding of the efficiency and behavior of these propulsion systems under different circumstances.

- Total temperature to static temperature ratio:

$$\frac{T_0}{T} - 1 + \frac{\gamma-1}{2} M^2 \tag{13}$$

- Temperature ratio across a normal shock:

$$\frac{T_2}{T_1} - \frac{\left(2\gamma M^2-(\gamma-1)\right)}{(\gamma+1)M^2} \tag{14}$$

- Oblique Shock Angle Calculation Solved using the implicit equation:

$$\tan(\theta) - \frac{2\cot(\beta)\left(M^2\sin^2(\beta)-1\right)}{M^2(\gamma+\cos(2\beta))+2} \tag{15}$$

- Oblique Shock Properties: Post-shock Mach number:

$$M_2 - \sqrt{\frac{\left(M_n^2+\frac{2}{\gamma-1}\right)}{\left(\frac{2\gamma M_1^2}{\gamma+1}-\frac{\gamma-1}{\gamma+1}\right)}} \tag{16}$$

- Pressure ratio:

$$\frac{P_2}{P_1} - 1 + \frac{2\gamma}{\gamma+1}\left(M_n^2 - 1\right) \tag{17}$$

- Temperature ratio:

$$\frac{T_2}{T_1} - \frac{\frac{P_1}{P_1}\left(2+(\gamma-1)M_n^2\right)}{(\gamma+1)M_n^2} \tag{18}$$

- Efficiency formula:

$$\eta - 1 - \frac{\left(\frac{T_n}{T_C}\right)-\left(P_{03}/P_{0A}\right)^{(\gamma-1)/\gamma}}{\left(\frac{T_n}{T_C}\right)-1} \tag{19}$$

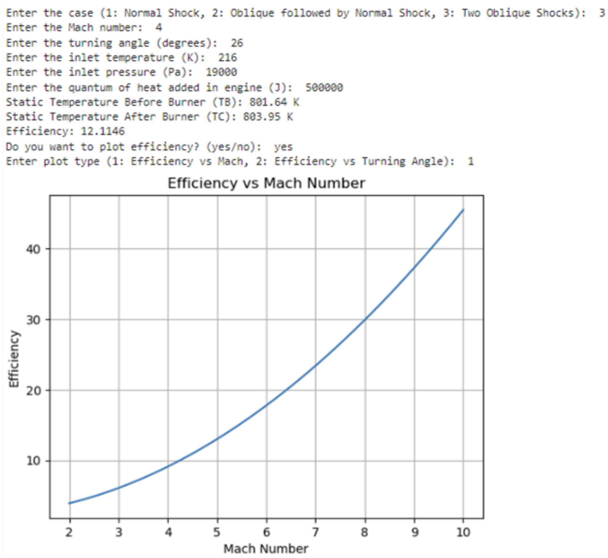

Fig 9. Code result and run for efficiency against mach number.

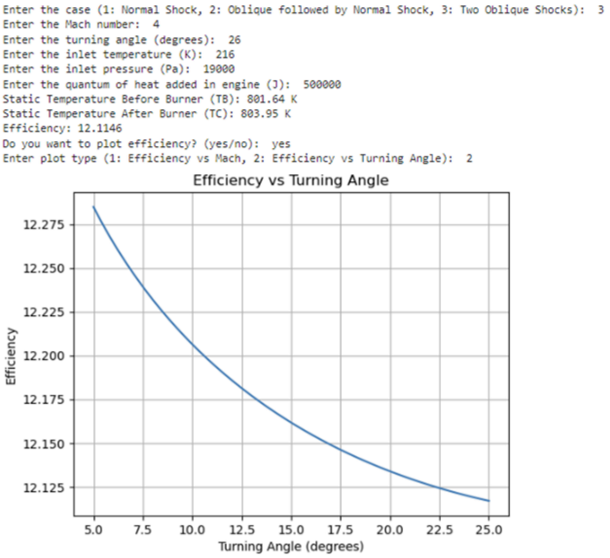

Fig. 10. Code result and run for efficiency against turning angle.

3. CONCLUSIONS

Utilizing an oblique shock and including gradual pressure recovery in the flow greatly improves the effectiveness of ramjets, as indicated by the analysis. The enhanced efficiency of systems using advanced shock management techniques becomes apparent when comparing them to traditional ramjets. More precisely, efficiency can be almost doubled, demonstrating the potential advantages of employing more advanced techniques to manipulate shock waves and gradually recover pressure. Moreover, scramjets provide an inherent advantage in terms of efficiency when compared to ramjets. This is because scramjets may operate efficiently at higher Mach numbers without requiring substantial slowing of the airflow prior to combustion. Nevertheless, this benefit is accompanied with heightened technological difficulties, including the management of elevated temperatures and the attainment of consistent combustion at supersonic velocities. The given data visually represents the influence of the angle of turn and the Mach number on the efficiency of a ramjet engine. The data illustrates that as the turning angle increases, the efficiency falls, indicating the engine performance's susceptibility to changes in the shock wave angle. The link between the turning angle and performance is crucial in the design of efficient propulsion systems, as reducing the turning angle can result in significant performance improvements. However, when Mach numbers climb, efficiency also increases, indicating the improved performance capability of ramjets and scramjets at higher speeds. This tendency highlights the significance of controlling operational circumstances in order to realize the advantages of high-speed airbreathing propulsion systems. In summary, the study underscores the need of utilizing sophisticated design and engineering methods to maximize the capabilities of scramjet technology. It also highlights the substantial enhancements that may be achieved by effectively controlling shock waves and optimizing pressure recovery in ramjet engines.

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APPENDIX- Python code

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import fsolve
def isentropic_temperature_ratio(mach, gamma=1.4):
    return 1 + (gamma - 1) / 2 * mach**2
def normal_shock_temperature_ratio(mach,
gamma=1.4):
    numerator = (2 * gamma * mach**2 - (gamma - 1))
    denominator = (gamma + 1) * mach**2
    return numerator / denominator
def oblique_shock_angle(mach, turning_angle,
gamma=1.4):
    turning_angle_rad = np.radians(turning_angle)
    func = lambda beta: np.tan(turning_angle_rad) - 2 *
np.tan(beta) * (mach**2 * np.sin(beta)**2 - 1) /
(mach**2 * (gamma + np.cos(2 * beta)) + 2)
    beta_initial_guess = np.arcsin(1 / mach) +
np.radians(turning_angle)
    beta = fsolve(func, beta_initial_guess)[0]
    return beta
def oblique_shock_properties(mach, turning_angle,
gamma=1.4):
    beta = oblique_shock_angle(mach, turning_angle,
gamma)
    mach_normal = mach * np.sin(beta)
    mach2 = np.sqrt((mach_normal**2 + 2 / (gamma -
1)) / ((2 * gamma * mach_normal**2 - (gamma - 1)) /
(gamma + 1)))
    pressure_ratio = (1 + 2 * gamma / (gamma + 1) *
(mach_normal**2 - 1))
    temperature_ratio = pressure_ratio * (2 + (gamma -
1) * mach_normal**2) / ((gamma + 1) *
mach_normal**2)
    return beta, mach2, pressure_ratio,
temperature_ratio
def efficiency(TC, TB, P03_P0A, gamma=1.4):
    return 1 - ((TB / TC) - (P03_P0A)**((gamma - 1) /
gamma)) / ((TB / TC) - 1)
def calculate_efficiency(case, mach, theta1=0,
theta2=0, T_inlet=0, P_inlet=0, heat_added=0,
gamma=1.4):
    if case == 1:
        # Normal Shock
        T0 = T_inlet *
isentropic_temperature_ratio(mach, gamma)
        TB = T0 /
normal_shock_temperature_ratio(mach, gamma)
        TC = TB + heat_added / (1004.5 * T_inlet)
        P03_P0A = 1 /
normal_shock_temperature_ratio(mach, gamma)
        eta = efficiency(TC, TB, P03_P0A, gamma)
        return TB, TC, eta
    elif case == 2:
        # Oblique Shock followed by Normal Shock
        beta1, mach2, _, _ =
oblique_shock_properties(mach, theta1, gamma)
```

```
        T0 = T_inlet *
isentropic_temperature_ratio(mach, gamma)
        TB = T0 /
normal_shock_temperature_ratio(mach2, gamma)
        TC = TB + heat_added / (1004.5 * T_inlet)
        P03_P0A = 1 /
normal_shock_temperature_ratio(mach2, gamma)
        eta = efficiency(TC, TB, P03_P0A, gamma)
        return TB, TC, eta
    elif case == 3:
        # Two Oblique Shocks
        beta1, mach2, _, _ =
oblique_shock_properties(mach, theta1, gamma)
        beta2, mach3, _, _ =
oblique_shock_properties(mach2, theta2, gamma)
        T0 = T_inlet *
isentropic_temperature_ratio(mach, gamma)
        TB = T0 /
normal_shock_temperature_ratio(mach3, gamma)
        TC = TB + heat_added / (1004.5 * T_inlet)
        P03_P0A = 1 /
normal_shock_temperature_ratio(mach3, gamma)
        eta = efficiency(TC, TB, P03_P0A, gamma)
        return TB, TC, eta
def plot_efficiency_vs_mach(case, theta1, theta2,
T_inlet, P_inlet, heat_added):
    mach_numbers = np.linspace(2, 10, 100)
    efficiencies = []
    for mach in mach_numbers:
        _, _, eta = calculate_efficiency(case, mach, theta1,
theta2, T_inlet, P_inlet, heat_added)
        efficiencies.append(eta)
    plt.plot(mach_numbers, efficiencies)
    plt.xlabel('Mach Number')
    plt.ylabel('Efficiency')
    plt.title('Efficiency vs Mach Number')
    plt.grid(True)
    plt.show()
def plot_efficiency_vs_turning_angle(case, mach,
T_inlet, P_inlet, heat_added):
    turning_angles = np.linspace(5, 25, 100)
    efficiencies = []
    for theta in turning_angles:
        _, _, eta = calculate_efficiency(case, mach, theta,
theta, T_inlet, P_inlet, heat_added)
        efficiencies.append(eta)
    plt.plot(turning_angles, efficiencies)
    plt.xlabel('Turning Angle (degrees)')
    plt.ylabel('Efficiency')
    plt.title('Efficiency vs Turning Angle')
    plt.grid(True)
    plt.show()
def main():
    case = int(input("Enter the case (1: Normal Shock,
2: Oblique followed by Normal Shock, 3: Two Oblique
Shocks): "))
    if case == 1:
        mach = float(input("Enter the Mach number: "))
        T_inlet = float(input("Enter the inlet temperature
(K): "))
        P_inlet = float(input("Enter the inlet pressure
(Pa): "))
        heat_added = float(input("Enter the quantum of
heat added in engine (J): "))
        TB, TC, eta = calculate_efficiency(case, mach,
T_inlet=T_inlet, P_inlet=P_inlet,
heat_added=heat_added)
        print(f"Static Temperature Before Burner (TB):
{TB:.2f} K")
        print(f"Static Temperature After Burner (TC):
{TC:.2f} K")
        print(f"Efficiency: {eta:.4f}")
    elif case == 2:
        mach = float(input("Enter the Mach number: "))
        theta1 = float(input("Enter the first turning angle
(degrees): "))
        T_inlet = float(input("Enter the inlet temperature
(K): "))
        P_inlet = float(input("Enter the inlet pressure
(Pa): "))
        heat_added = float(input("Enter the quantum of
heat added in engine (J): "))
        TB, TC, eta = calculate_efficiency(case, mach,
theta1=theta1, T_inlet=T_inlet, P_inlet=P_inlet,
heat_added=heat_added)
        print(f"Static Temperature Before Burner (TB):
{TB:.2f} K")
        print(f"Static Temperature After Burner (TC):
{TC:.2f} K")
        print(f"Efficiency: {eta:.4f}")
    elif case == 3:
        mach = float(input("Enter the Mach number: "))
        theta = float(input("Enter the turning angle
(degrees): "))
        T_inlet = float(input("Enter the inlet temperature
(K): "))
        P_inlet = float(input("Enter the inlet pressure
(Pa): "))
        heat_added = float(input("Enter the quantum of
heat added in engine (J): "))
        TB, TC, eta = calculate_efficiency(case, mach,
theta1=theta, theta2=theta, T_inlet=T_inlet,
P_inlet=P_inlet, heat_added=heat_added)
        print(f"Static Temperature Before Burner (TB):
{TB:.2f} K")
        print(f"Static Temperature After Burner (TC):
{TC:.2f} K")
        print(f"Efficiency: {eta:.4f}")
    plot_choice = input("Do you want to plot efficiency?
(yes/no): ")
    if plot_choice.lower() == 'yes':
        plot_type = int(input("Enter plot type (1:
Efficiency vs Mach, 2: Efficiency vs Turning Angle):
"))
        if plot_type == 1:
```

```
        plot_efficiency_vs_mach(case, theta1 if case == 2 else 0, theta if case == 3 else 0, T_inlet, P_inlet, heat_added)
    elif plot_type == 2:
        plot_efficiency_vs_turning_angle(case, mach, T_inlet, P_inlet, heat_added)
if __name__ == "__main__":
    main()
```